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A stable and convergent Lagrangian particle method with multiple nodal stress points for large strain and material failure analyses in manufacturing processes

C.T. Wu [a,*], Y. Wu [a], Z. Liu [a], D. Wang [b]

[a] Livermore Software Technology Corporation (LSTC), Livermore, CA 94551, USA
[b] Department of Civil Engineering, Xiamen University, Xiamen, Fujian, 361005, China

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ABSTRACT

This paper presents a new Lagrangian particle method for the simulation of manufacturing processes involving large strain and material failure. The starting point is to introduce some stabilization terms as a means of circumventing the onerous zero-energy deformation in the Lagrangian particle method. The stabilization terms are derived from the approximate strain vector by the combination of a constant and strain derivatives, which leads to a *multiple nodal stress points* algorithm for stabilization. The resultant stabilized Lagrangian particle formulation is a non-residual type that renders no artificial control parameters in the stabilization procedure. Subsequently, the stabilized formulation is supplemented by an adaptive anisotropic Lagrangian kernel and a bond-based material failure criterion to sufficiently prevent the tension instability and excessive straining problems. Several numerical examples are presented to examine the effectiveness and accuracy of the proposed method for modeling large strain and material failure in manufacturing processes.

1. Introduction

The advanced numerical simulation of large strain and material failure problems is in great demand for manufacturing engineers and has become an important research topic in computational solid mechanics. It is recognized that the Lagrangian finite element method (FEM) has strong limitations in simulating this type of problems due to severe mesh distortion. Although there is no mesh distortion in the Eulerian FEM, the Eulerian approach has considerable difficulty to trace the moving interface in material failure analyses because the position of a material point relative to the Eulerian node is varying with the motion. The arbitrary Lagrangian-Eulerian (ALE) [1] method is another mesh-based numerical approach in which the computational system is neither attached to the material nor prior-fixed in space. Since the computational mesh inside the domain is moving arbitrarily to optimize the shape of elements, the ALE algorithm is also hard to advance the moving interface accurately in material failure analyses particularly for three-dimensional problems.

Particle methods, or sometimes called the "meshfree" methods have attracted significant attention from scientists and engineers over the last two decades in modeling challenging scientific and engineering problems. Specially, the Lagrangian particle methods have demonstrated their superiority over the standard mesh-based numerical methods in large strain [2–4], material failure [5–7] and immersed [8–10] problems. Lagrangian particle methods were also found to be very effective on reducing volumetric locking and shear locking in solid and structural analyses [11–13]. Smoothed Particle Hydrodynamics (SPH) method, initially developed by Gingold and Monaghan [14] and Lucy [15], is commonly considered the earliest Lagrangian particle method in computational mechanics. Despite its success in simulating the free-surface flow and high-velocity impact problems [16–18], SPH is known to experience several numerical instabilities, for example, the tensile instability [19], spurious zero-energy mode [20] and excessive straining [21] in solid mechanics applications.

Intensive research has been done to resolve some of those numerical instabilities. For instance, the introduction of Lagrangian kernel [2,22] and stress points method [23] have been proven to effectively remove the tension instability in many Lagrangian particle methods, regardless of their restricted ability to handle severe deformation. Spurious zero-energy mode is the numerical instability caused by rank deficiency of the particle discrete system using the direct nodal integration scheme [20]. The approach to stabilize particle methods is fundamentally different from finite element nodal integration methods due to the

* Corresponding author.
E-mail address: ctwu@lstc.com (C.T. Wu).

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rational characteristic of approximation functions. Various particle stabilization approaches such as the residual-type and non-residual type stabilization methods [24–26], Stabilized Conforming Nodal Integration (SCNI) method [27] and variationally consistent integration methods [28,29] have been proposed to suppress the spurious zero-energy mode. In the literature [21,30,31], the term *excessive straining* is commonly applied to describe the numerical difficulty in large strain analyses when the strictly use of the Lagrangian kernel [2] is no more applicable. The excessive straining leads to a system of equations that ceases to become invertible and stable. The excessive straining could combine with tension instability and spurious zero-energy mode giving another instance of unstable solution in Lagrangian particle methods, which is very problematic. Excessive straining also appears in material failure analysis. Specifically, the C^1-continuity assumption in most Lagrangian particle methods is inadequate to describe the kinematic discontinuity of displacement field in a continuous setting for failure analysis. This makes Lagrangian particle methods challenging in material failure analysis. The extant literature in Lagrangian particle methods [5,21,32,33] gives very few clues about this numerical issue. There is also an ambiguity in dealing with the interaction of particles that are damaged or failed in the material failure analysis. Some works allow the non-physical material self-healing (fusion) through the kernel reconstruction, while others simply neglect the contact between failed particles. To the authors' best knowledge, very few studies [29,34] have addressed these numerical issues comprehensively, and satisfactory solution in large strain and material failure analyses is not well documented numerically either.

The Smoothed Particle Galerkin (SPG) method [34] is one of the few Lagrangian particle methods develop recently to deal with those numerical instabilities in solid mechanics applications. The essence of SPG method is to augment the standard quadratic energy functional by a non-residual term, which leads to a stabilized variational formulation [26] for stabilization. Another stabilized Lagrangian particle method was proposed by Hillman and Chen [29] for the severe deformation analysis. In their method, the strain gradient stabilization technique [35,36] was chosen to give sufficient control on zero-energy modes. Furthermore, an implicit gradient expansion [28] was introduced to avoid the calculation of high-order derivatives in stabilization terms. As opposed to the conventional residual-type stabilization method [24] which uses residuals of the momentum equations and artificial control parameters to effect stabilization, these new stabilization methods are considered the penalty-based h^2-stabilization formulations [26] whose purpose is to bypass the dependence of artificial control parameters for stabilization. In order to enable the Lagrangian kernel in severe deformation analysis, either the semi-Lagrangian kernel [30] or the adaptive anisotropic Lagrangian kernel [31] has been employed in these stabilized formulations. In comparison to the conventional finite element method, the integration of weak form of those new stabilization methods is still time-consuming in small deformation applications, mainly due to the large number of interacting particles and the inefficient neighbor sorting. However, their robustness and accuracy in severe deformation analysis are in general remarkably higher than the corresponding finite element method with the same order of approximation and discretization. Aside from superior performance in large strain analysis, the application of those stabilized Lagrangian particle methods to material failure analysis needs to be further investigated.

In this paper, a new stabilized Lagrangian particle method, motived by the stabilized variational formulation [26,31] and strain gradient stabilization technique [29,35,36], is presented for large strain and material failure analyses. The present method of stabilization involves multiple nodal stress points for integration but does not require specification of a stabilization parameter. To sufficiently prevent the tension instability and excessive straining in large strain and material failure simulations, the proposed stabilized formulation is supplemented with the adaptive anisotropic Lagrangian kernel [31] and the bond-based failure criterion [34]. The rest of the paper is outlined as follows. In Section 2, an overview on stabilized variational formulation for stabilization is provided. In Section 3, a *multiple nodal stress points* algorithm is described. The numerical implementation is given in Section 4. Various numerical examples are presented in Section 5. Final remarks are given in Section 6.

2. Overview on the stabilized variational formulation in linear elasticity

Consider a homogeneous isotropic linear elastic material body which occupies a bounded domain Ω in $\mathbb{R}^3$ with Lipschitz boundary Γ. For simplicity, the homogenous Dirichlet problem [37] is assumed in the following variational derivation. The admissible space for the displacement fields is defined by

$$\boldsymbol{V}(\Omega) = \left\{ \boldsymbol{v} : \boldsymbol{v}|_{\Omega} \in \boldsymbol{H}^1(\Omega), \boldsymbol{v} = \boldsymbol{0} \ \ on \ \ \Gamma \right\} \tag{1}$$

where $\boldsymbol{H}^1(\Omega)$ denotes the Sobolev space of degree one.

The general form of stabilized variational (PV) formulation for the stabilization in particle methods reads as [26]:

$$(PV): \begin{cases} find \ \ \boldsymbol{u} \in \boldsymbol{V} \ \ such \ \ that \ \ \tilde{\Pi}(\boldsymbol{u}) = \inf\limits_{\boldsymbol{v} \in \boldsymbol{V}} \tilde{\Pi}(\boldsymbol{v}) \\ \\ \tilde{\Pi}(\boldsymbol{v}) = \dfrac{1}{2} A(\boldsymbol{v}, \boldsymbol{v}) + \dfrac{1}{2} Q(\boldsymbol{v}, \boldsymbol{v}) - l(\boldsymbol{v}) \end{cases} \tag{2}$$

In linear elasticity problem, the standard bilinear form $A(.,.)$ and linear functional $l(.)$ are defined by

$$A : \boldsymbol{V} \times \boldsymbol{V} \rightarrow \mathbb{R}, \quad A(\boldsymbol{v}, \boldsymbol{v}) := \int_{\Omega} \boldsymbol{\varepsilon}(\boldsymbol{v}) : \boldsymbol{C} : \boldsymbol{\varepsilon}(\boldsymbol{v}) d\Omega \tag{3}$$

$$l : \boldsymbol{V} \rightarrow \mathbb{R}, \quad l(\boldsymbol{v}) := \int_{\Omega} \boldsymbol{b} \cdot \boldsymbol{v} d\Omega \tag{4}$$

where $\boldsymbol{b}$ is the prescribed body force, $\boldsymbol{\varepsilon}(\boldsymbol{v})$ is the infinitesimal strain tensor which is a function of the displacement $\boldsymbol{v}$, and $\boldsymbol{C}$ is the fourth-order elasticity tensor. The bilinear form $Q(.,.)$ is defined by Ref. [26].

$$Q(\boldsymbol{v}, \boldsymbol{v}) = \frac{1}{2} \int_{\Omega} \left(\overline{\Theta} \boldsymbol{\varepsilon}(\boldsymbol{v}) - \boldsymbol{\varepsilon}(\boldsymbol{v}) \right) : \boldsymbol{C} : \left(\overline{\Theta} \boldsymbol{\varepsilon}(\boldsymbol{v}) - \boldsymbol{\varepsilon}(\boldsymbol{v}) \right) d\Omega \tag{5}$$

which is the stabilization term introduced to enhance the coercivity of $A(.,.)$. Obviously, the assumption on $\boldsymbol{C}$ guarantees that the stabilization term $Q(.,.)$ is symmetric and positive semi-definite. Mathematically, the term $Q(\boldsymbol{v}, \boldsymbol{v})$ describes an interior penalty associated with the constraint $\overline{\Theta} \boldsymbol{\varepsilon}(\boldsymbol{v}) - \boldsymbol{\varepsilon}(\boldsymbol{v}) = 0$ where $\overline{\Theta} : L^2(\Omega) \rightarrow L^2(\Omega)$ is a gradient projection operator applied to tensor $\boldsymbol{\varepsilon}(\boldsymbol{v})$ component-wise.

For a particle distribution denoted by an index set $Z_I = \{\boldsymbol{X}_I\}_{I=1}^{NP} \subset \mathbb{R}^3$, approximating the displacement field using the meshfree approximation gives

$$\boldsymbol{u}^h(\boldsymbol{X}, t) = \sum_{I \in Z_I} \varphi_I^a(\boldsymbol{X}) \boldsymbol{u}(\boldsymbol{X}_I, t) = \sum_{I \in Z_I} \varphi_I^a(\boldsymbol{X}) \tilde{\boldsymbol{u}}_I(t) \equiv \widehat{\boldsymbol{u}}(\boldsymbol{X}, t) \quad \forall \boldsymbol{X} \in \Omega^0 \tag{6}$$

where NP is the total number of particles in discretization. $\varphi_I^a(\boldsymbol{X})$, $I = 1, \ldots, NP$, can be interpreted as the Lagrangian shape functions of the meshfree approximation for displacement field $\boldsymbol{u}^h$ where the superscript "a" denotes the radius size of $\varphi_I^a(\boldsymbol{X})$.

In Eq. (6) $\tilde{\boldsymbol{u}}_I(t) := \boldsymbol{u}(\boldsymbol{X}_I, t)$ is called the "generalized displacement" [2] of particle I in Galerkin meshfree method since the Kronecker-delta property generally no longer holds in meshfree approximation. In other words, the material displacement $\boldsymbol{u}^h(\boldsymbol{X}, t)$ is considered as an interpolant of $\boldsymbol{u}(\boldsymbol{X}_I, t)$ in a generalized sense. As a result, a special essential boundary condition treatment is needed [2,38]. In order to avoid the complexity in the enforcement of essential boundary condition, a first-order meshfree convex approximation [39] is considered. In this study, the meshfree convex approximation is constructed by the Generalized Meshfree Approximation (GMF) method [39]. With the meshfree convex approximation, one can define the $\boldsymbol{H}_0^1$-conforming subspace for the