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Use of stochastic XFEM in the investigation of heterogeneity effects on the tensile strength of intermediate geotechnical materials

Ashley P. Dyson [a], Zhan Tang [b], Ali Tolooiyan [a,*]

[a] *Geotechnical and Hydrogeological Engineering Research Group (GHERG), Federation University Australia, Gippsland Campus, Victoria, Australia*
[b] *Faculty of Science, Monash University, Australia*

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ABSTRACT

The numerical simulation of an Unconfined Expansion Test (UET) is presented with tensile strength fracture criteria assigned by stochastic methods to take into account material heterogeneity. Tests are performed by producing radial cavity expansion models of thinly sliced cylindrical specimens. The introduction of element-wise allocation of fracture parameters generates instances of specimen failure without the requirement of predefined fracture zones, permitting discontinuities to form naturally within zones containing weak strength parameters. The parallel application of an in-house Python scripts and eXtended Finite Element Method (XFEM) facilitates the investigation of heterogeneity effects on the tensile strength of intermediate geotechnical materials.

1. Introduction and background

Conventional numerical modelling methods such as the Finite Element Method (FEM) often perform poorly in approximation of solutions with non-smooth characteristics in the modelling domain, for example near discontinuities, crack initiation/propagation, and singularities. The eXtend Finite Element Method (XFEM) developed by Belytschko and Black [1] is an effective method to simulate discontinuities and crack opening by enabling a local enrichment of approximation spaces. With the additional degrees of freedom from special enrichment functions, once the failure criteria are fulfilled, cracks are allowed to initiate in local enrichment regions and to propagate based on the energy release criteria without the need for conventional re-meshing. Due to the absence of remeshing requirements, XFEM has become one of the choice methods for modelling fracture of cohesive materials [2–5].

Theoretically, the simplest laboratory testing method for the measurement of tensile strength is the Direct Tension Test (DTT). The method has been extensively implemented in the investigation of the tensile strength of over-consolidated clays, unsaturated soils, and cemented sands. However, the complicated process of specimen preparation is a significant obstacle [6,7]. Coupled with potential bias when compared with the Brazilian Testing method [8], Tang et al. [9] developed an XFEM model using Abaqus code [10] to simulate crack opening and tension failure in Unconfined Expansion Testing (UET) in order to analyse both the stress distribution of the test and to examine fracture parameters. UET (as seen in Fig. 1) is a newly designed test that aims at measuring the tensile strength of intermediate geotechnical materials (IGM) such as soft rocks. A cylindrical cavity is drilled along the axis of a cylindrical specimen, then, based on Timoshenko's thick wall cylinder expansion theory [11] the UET method is able to create circumferential, uniformly-distributed tensile stresses around the sample cavity by the inflation of an expandable probe. Therefore, an arbitrary crack path is created when natural weaknesses of the sample approach the material's tensile strength. XFEM simulation results confirm an agreement with theoretical assumptions in both pre-failure stress distribution and tensile strength [9]. As stresses evenly develop inside the geometry, a radial XFEM region must be predefined within the geometry to overcome the computational ambiguity of crack opening when XFEM is specified over the full domain.

Of particular interest in the study of UETs is the impact of material heterogeneity. This can be achieved by interpolating XFEM with material variability and performing probabilistic analyses. Probabilistic analysis of heterogeneous brittle materials has continued to gain significant attention [2,12,13]. Understanding of the relationship between heterogeneity and UET test results has two benefits. For materials with known heterogeneity, the method can be applied to determine the test quality and possible error. For unknown heterogeneous materials, engineers can back-estimate the material's heterogeneity by performing a large number

* Corresponding author.
E-mail addresses: a.dyson@federation.edu.au (A.P. Dyson), zhan.tang@monash.edu (Z. Tang), ali.tolooiyan@federation.edu.au (A. Tolooiyan).

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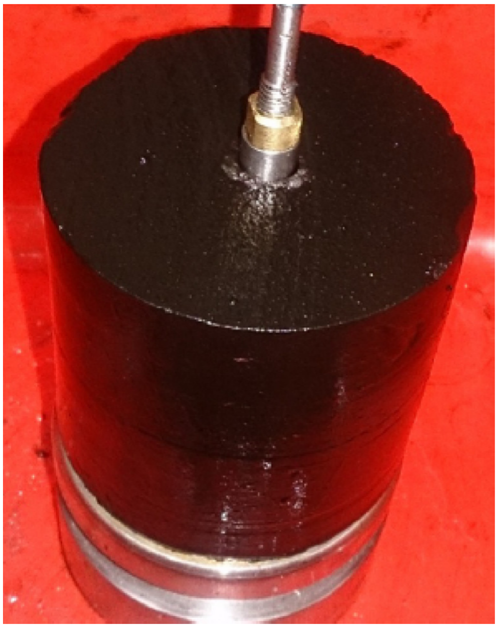

Fig. 1. UET probe placed within a brown coal specimen, Tang et al. [9].

of tests.

This paper introduces the existing uniform XFEM simulation of UET but without a predefined fracture zone. Commencing with the theory of discontinuity simulation by XFEM for implementation of UET modelling, comparisons of scenarios with and without the predefined XFEM region are produced. The process of random variable parameter assignment is detailed, followed by XFEM simulation results. With the aforementioned approach, the goal of this research is to allow fracture to form naturally without predefined XFEM zones, by the process of random variable assignment of element parameters. Furthermore, this paper presents the effects of this process on material strength, such that stochastic models incorporating variation of material characteristics can be calibrated to align with observed laboratory UET results, while also producing numerical results to develop conclusions about empirical UET parameter variability.

2. The extended finite element method overview

The extended finite element method is an indispensable tool for modern numerical simulation of crack initiation, propagation, and coalescence. Initially developed by Belytschkoand Black [1,15], XFEM supplements the classic finite element method with local enrichment functions applied to finite element approximation spaces. Alternative existing techniques for fracture simulation include boundary element methods [16,17], continual re-meshing FEM [18], and mesh-free methods [14,19]. However, XFEM exhibits a wide range of beneficial attributes; without the enrichment properties of the extended method, conventional FEM requires appreciable mesh refinement in the neighbourhood of discontinuity tips. Discrete crack propagation phenomena are often modelled efficiently with XFEM without the constraint of extensive re-meshing, a crucial property for computationally expensive non-linear systems. XFEM incorporates the added benefit of integrating readily available codes to existing finite element algorithms and framework.

2.1. Theory

In the extended finite element method, the span of functions for the element-free Galerkin method of Fleming, Chu [19] is implemented utilising Partition of Unity (PU) theory developed by Melenk and Babuška [20], and Duarte and Oden [21]. The PU method provides analysis of material behaviour characteristics throughout element geometries, rather than exclusively at element nodes. Consequently, mesh and discontinuity alignments are nonessential. Additional degrees of freedom produce near crack tip nodes by enrichment functions. The displacement approximation u, with the partition of unity enrichment is expressed in Equation (1) [10].

$$\boldsymbol{u}^h = \sum_{i\in I}\boldsymbol{u}_i\phi_i + \sum_{j\in J}\boldsymbol{b}_j\phi_j H(x) + \sum_{k\in K}\phi_k\left(\sum_{l=1}^{4}\boldsymbol{c}_k^l F_l(\boldsymbol{x})\right) \tag{1}$$

where the first contributing part $\mathbf{u}_i$ corresponds to the classical FEM approximation of the displacement field, φ_j is the shape function for the jth node whose support is cut by the crack face (but not the crack tip), $\mathbf{b}_j$ is the crack face jth nodal displacement vector; *H(x)* is the modified Heaviside function (Equation (2)), $F_l(x)$ are the elastic asymptotic crack tip functions (Equations (3)–(6)); $\mathbf{c}^l_k$ is kth nodal enriched degree of freedom vector of the asymptotic crack tip and φ_k is the shape function for the kth node whose support is cut by the crack tip. The nodal displacement term $\mathbf{u}_i$ applies to all nodes, while the Heaviside term contributes only to nodes whose support is cut by the crack interior. The final term of Equation (1) is applied only to the nodes whose support is cut by the crack tip.

$$H(\boldsymbol{x}) = \begin{cases} 1 & \text{if } (\boldsymbol{x}-\boldsymbol{x}^*)\cdot\boldsymbol{m} \geq 0, \\ -1 & \text{otherwise} \end{cases} \tag{2}$$

where, $\boldsymbol{x}$ is a Gauss point, $\boldsymbol{x}^*$ is the projection of $\boldsymbol{x}$ onto the crack line, and $\boldsymbol{m}$ is the unit outward normal to the crack at position $\boldsymbol{x}^*$.

$$F_{l=1}(x) = \sqrt{r}\sin\frac{\theta}{2} \tag{3}$$

$$F_{l=2}(x) = \sqrt{r}\cos\frac{\theta}{2} \tag{4}$$

$$F_{l=3}(x) = \sqrt{r}\sin\frac{\theta}{2}\sin\theta \tag{5}$$

$$F_{l=4}(x) = \sqrt{r}\cos\frac{\theta}{2}\sin\theta \tag{6}$$

Where, (r, θ) form the polar coordinate system with the crack tip at the origin.

2.2. Extended finite element simulations with predefined enrichment zones

The numerical fracture simulations of Tang et al. [9] provides an innovative technique for the modelling of the tensile strength of organic soft rock cored specimens. Utilising the XFEM in Abaqus 6.14 FEM code, Tang et al. [9] produced a quarter annulus shaped membrane (Fig. 2) with spatially homogeneous material properties shown in Table 1. As shown in Fig. 2, a predefined thin enrichment XFEM region is extending radially from a centre cavity. Due to the expectation of stress developing uniformly through the model geometry, crack initiation and propagation are exclusively allowed to develop within the desired XFEM region.

To trigger the crack initiation, the failure criterion was defined by a Maximum Principal Stress Failure Criteria (MPSFC) of 130 kPa, suitably modelling failure under tension. Conversely, the deviatoric stress criterion modelled failure in compression, as expected in elements close to the annulus cavity wall, where aperture expansion may cause excessive compression.

In preference to implementing a computationally expensive full three-dimensional model, a thin layer consisting of 3842 eight-node linear brick reduced integration elements (ABAQUS type C3D8R) was deemed a suitable specimen geometry for initial analysis. A thin 3D layer was chosen in preference to a 2D plane-strain model, due to the contact