



Improving mass matrix and inverse mass matrix computations of hexahedral elements

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ABSTRACT

We employ a semi-analytical approach to derive new practical schemes for mass matrix computation of 8-node and 20-node hexahedral elements. The new schemes offer accuracy equivalent to that of the conventional numerical integration (quadrature rule) with a significantly smaller number of integration points. Specifically, for the 8-node hexahedral element, we propose a 4-point rule to replace the currently used 8-point quadrature. Also, for the 20-node hexahedral element, we propose a 4-point scheme to replace the 14-point quadrature adopted by ANSYS and a 10-point scheme to replace the 27-point quadrature adopted by ABAQUS. In addition, we develop a novel approach for direct computation of the inverse mass matrix of 8-node hexahedral elements. This new approach requires a computational effort equivalent to standard numerical integration and eliminates the high computational cost associated with matrix inversion.

1. Introduction

The demand for simulating the response of increasingly larger structures has led to a continuous effort over the last decades to reduce the computation time of finite-elements (FE) analyses. In parallel to the development of faster processors and more powerful hardware, efforts have also been made in developing more efficient numerical schemes and new element types. For example, a 10-node tetrahedral element that does not exhibit numerical locking, even for incompressible materials, was recently developed by Jabareen et-al [1] based on a closed-form formulation of a Cosserat-Point Element. Other examples are the use of symbolic computations along with code generation to reduce the computation time of numerical integration procedures (e.g. Refs. [2–5]). Closed form integration has also been employed in improving the efficiency of stiffness-matrix calculation for specific elements, such as plane elements [6–9], triangles [10,11], 3-D bricks [7,12], and 3-D tetrahedral elements [11,13–17]. Other studies developed a fast method of numerical quadrature for p-version finite element matrices [18], or adopted a systematic approximation combined with closed-form integration to obtain closed-form expressions for the natural frequencies of skewed parallelepipeds [19].

All commercial FE packages and vast majority of FE analyses adopt a numerical integration procedure to evaluate the mass matrices (either consistent, lumped or in some cases their linear combination) of solid

elements. The common numerical integrator, quadrature, is based on evaluating the integrand at carefully chosen “integration points” [20,21]. The more integration points one uses, the higher accuracy follows. In a recent paper [22], we developed a new numerical scheme which can replace the standard numerical quadrature in computing mass matrices of solid elements. Similarly to the standard approach, more integration points lead to better accuracy, and the scheme converges to the exact mass matrix if the number of integration points is sufficiently large. In principle, the new approach is not limited to a specific element type, but in Ref. [22] it was specialized to the 10-node tetrahedral element. It was shown that, compared to standard quadrature with the same number of integration points, the new scheme significantly improves the accuracy of the numerical integration.

In the current contribution, we specialize the approach, first presented in Ref. [22], to the 8-node and 20-node hexahedral elements. We show that, compared to standard quadrature, the new scheme requires a significantly smaller number of integration points in order to provide the same level of accuracy, thus reducing the computation time. For example, our semi-analytical (SA) 4-point rule for the 8-node brick provides accuracy equivalent to that of 8-point standard quadrature (ST). Also, for the 20-node brick, our SA 4-point and 10-point schemes provide accuracy equivalent to that of the 14-point and 27-point standard quadrature, respectively.

In addition, we generalize the approach of [22] to enable the direct

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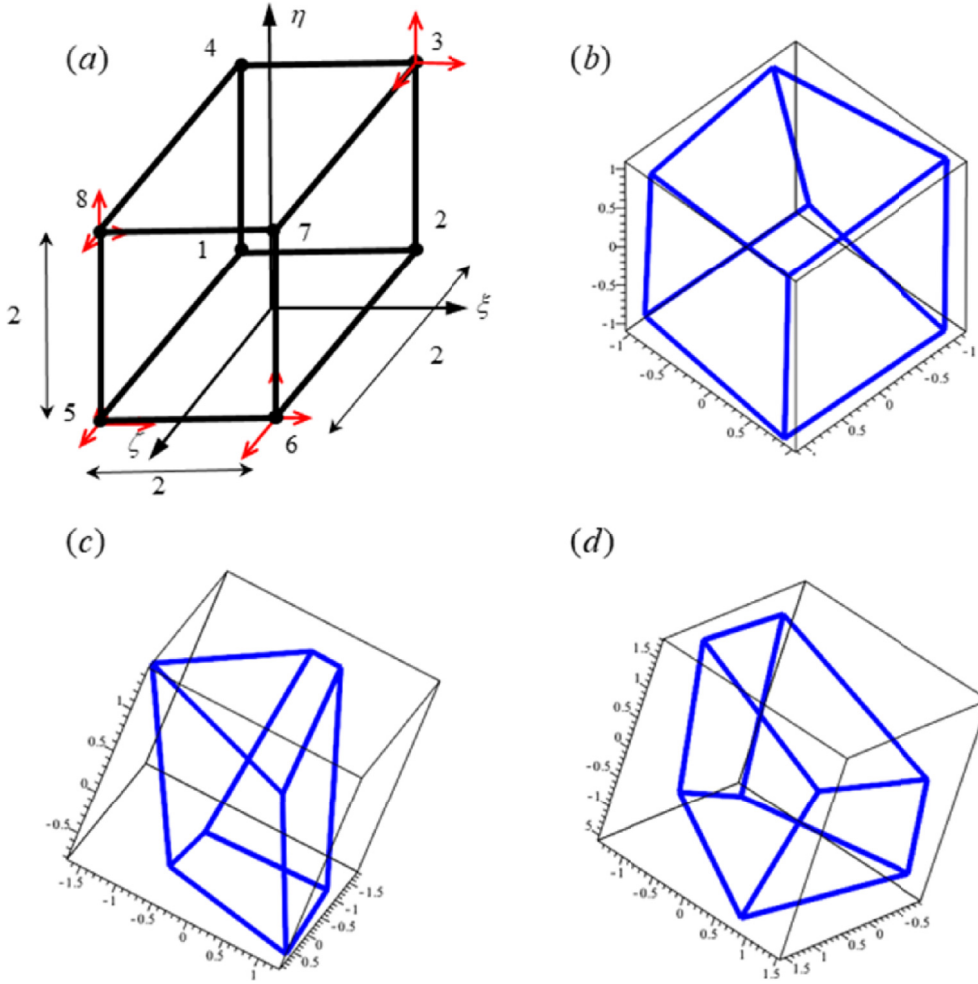


Fig. 1. Generating the learning set for the 8-node hexahedral element. (a) parent element. Relocation of nodes by random number is illustrated using arrows (several arrows are omitted for clarity). (b,c,d) examples of elements with $\delta = 0.16, 0.83, 1.0$, respectively.

computation of the *inverse* of the consistent mass matrix (with no need of inversion). In particular, we are able to compute the inverse of the consistent mass matrix with a computational effort similar to that of calculating the consistent mass matrix itself with standard quadrature. This unique feature has the potential to significantly improve accuracy and computation time in dynamic analysis, especially in cases where coarse-mesh elements are present. For example, in explicit dynamic analysis, lumped (diagonal) mass matrix is used in order to reduce the cost associated with solving a coupled set of equations. However, in some problems, diagonal mass matrices may introduce substantial errors [23]. It has been shown [24,25] that accuracy can be significantly improved while maintaining a reasonable computational cost by combining the use of the consistent mass matrix in highly deformed (or coarse mesh) regions with calculations based on an element-by-element technique [26], where the elements inverse mass matrices are employed instead of global assembly of the mass matrix. In Ref. [27] the element inverse mass matrix was approximated by an iterative scheme, in Ref. [25] the method of localized Lagrange multipliers was adopted, while a vibrational construction was considered in Ref. [28]. The direct computation of the element inverse mass matrix presented here can significantly reduce the computational cost associated with this step.

Following the above, the paper is organized as follows: Section 2 recalls the main theoretical considerations and mathematical formulation of the semi-analytical (SA) and optimization-based (OB) approaches. The latter is used for the direct computation of the element inverse mass matrix. Section 3 gives details, namely ansatz functions, integration points, and the coefficients matrices (generalized weights), to formulate specific ready-to-implement SA integration schemes for the 8-node and

20-node brick elements. Section 4 presents the details to formulate the OB scheme for the 8-node brick element. Numerical results, that examine the accuracy of the SA and the OB schemes, are presented in Section 5. Main conclusions are discussed in Section 6.

2. Theoretical considerations and background

In this section, we briefly recall the basic concepts and considerations behind the “semi analytical” (SA) method, first presented in Ref. [22], for calculating the element mass matrix. In addition, we present a new approach that enables direct calculation of the element *inverse* mass matrix. As discussed below, the computational effort of these schemes is equivalent to that of the conventional/standard rule for the same number of integration points.

The consistent mass matrix of a solid finite element having n_{nodes} nodes with shape functions $N_i, i = 1, \dots, n_{nodes}$, initial density and volume ρ_0 and V , respectively, is defined as $M_{ij} = \int_V N_i N_j \rho_0 dV$. In terms of local element coordinates ξ, η, ζ , it follows that

$$M_{ij} = \int_V N_i N_j (\rho_0 J) d\xi d\eta d\zeta, \quad (1)$$

where J is the metric (or Jacobian) of global to the local coordinate transformation [29]. Typically, this integral is numerically calculated by a standard (ST) numerical integration scheme which is based on evaluating the integrand at some carefully chosen “integration points”, i.e. $M_{ij} \approx M_{ij}^{ST} = \sum_{p=1}^{n_p} w_p N_{ip} N_{jp} \rho_{0p} J_p$. Here, n_p denotes the number of integration points, w_p are corresponding weights, and N_{ip}, ρ_{0p}, J_p represent

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