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A computational strategy for the modeling of elasto-plastic materials under impact loadings

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ABSTRACT

This paper presents a numerical strategy for the modeling of elasto-plastic materials under impact loadings. The bi-potential method is efficient in dealing with implicit standard materials constitutive laws, such as frictional contact. The Uzawa algorithm, based on the bi-potential method is applied to deal with the non-linearity of boundary conditions. For the non-linearity associated with materials, a Return Mapping Algorithm is applied to deal with the elasto-plastic constitutive laws. On the basis of the Updated Lagrangian formulation, we adopt a Rotationally Neutralized Objective hypothesis to describe the geometrically non-linear behavior. Equations of motion are integrated with a first order scheme. Three numerical examples are performed to verify the accuracy and to show the applicability of the proposed approach.

1. Introduction

Accurate modeling and simulation of elasto-plastic materials under impact loadings is always considered to be a complex problem in solid mechanics. In this paper, we propose a strategy to tackle this issue. From the mechanical point of view, this strategy can be separated into three main parts: frictional contact, material's non-linearity of finite strain and equations of motion [14].

Handling the non-linearity of boundary condition, which is the frictional contact problem, is of great importance in many engineering applications. It is certainly one of the top non-linear mechanics topics [17]. Literature shows that the finite element procedures are widely used to deal with such problems [12]. To establish the constitutive laws of frictional contact, many attempts have been made in the past. The penalty approximation, for example, is suitable for many applications [4]. However, in some cases, inappropriate penalty factor leads to inaccurate contact boundary conditions and friction laws [11]. Investigators promoted some strategies to improve the accuracy of the penalty method [2,19]. It is proved that the extended formulation of the augmented Lagrangian method has excellent performances in dealing with frictional contact issues [1,15]. On the basis of the augmented Lagrangian, a bi-potential method was promoted by de Saxcé and Feng, where dual variables satisfy the Fenchel's transform [3]. The bi-potential method not only combines Signorini conditions and Coulomb's friction laws, but is also more efficient to approach the constitutive laws of frictional contact. To solve the general frictional contact equations, the Uzawa algorithm targets only the global governing equation of contact, making it faster and more suitable than the Newton Algorithm, as verified by Joli and Feng [10]. Therefore, combining the bi-potential method with the Uzawa algorithm to solve frictional contact governing laws is a highly feasible strategy.

Regarding constitutive relationships, the material non-linearity is considered through a mixed J2 model. The finite and permanent deformations always occur during the impact process. The Return Mapping Algorithm (RMA) is a classic implicit algorithm put forward by Simo and Taylor, and is widely used in elasto-plasticity [16]. The RMA satisfies both the global equilibrium equations and the local material constitutive laws in every incremental step. It leads to a better stability during the solving process [14]. Under impact loadings, the constitutive relationships are always influenced by the geometrical non-linearity, plenty of hypothesis about the configurations have been promoted. The Total Lagrangian (TL) frame is usually used in hyper-elastic models. For elasto-plastic materials, many of them are described in an Updated Lagrangian (UL) frame. What is more, the Rotationally Neutralized Objectivity (RNO) is one hypothesis that proves to be stable and efficient for simulating large deformations of elasto-plastic materials [14].

Upon combining frictional contact, material non-linearity and large deformation, the governing equations of the elasto-plastic impact will be

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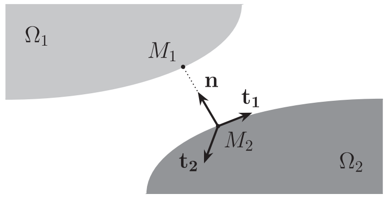

Fig. 1. The kinematics in the local contact system.

time-integrated. To avoid discontinuities in velocity and acceleration, a first order algorithm, proposed by Jean [8], is chosen. The method has successfully been verified on hyper-elastic materials by Feng et al. [6].

This work focuses on the application of an accurate modeling strategy in dealing with frictional contact laws of general elasto-plastic materials under impact loadings. As in Ref. [10], the Uzawa algorithm is implemented within the bi-potential framework to solve frictional contact equations. Meanwhile, on the foundation of the rotationally neutralized objective algorithm, the RMA is chosen to handle the material constitutive laws. Then, a first order algorithm is chosen to handle the time integration of motion equations. Our strategy combines these three aspects and is implemented into the in-house finite element software FEM/Form.

This paper is structured as follows. First, a brief presentation of the bi-potential method and the Uzawa algorithm in dealing with frictional contact will be given in Section 2. Then, Section 3 focuses on the implementation of elasto-plastic materials constitutive laws by applying the RMA on an classic mixed J2 model under the rotationally neutralized objective framework. To solve the highly non-linear governing equation, a first order algorithm is applied to integrate the equation of motion presented in Section 4. In addition, Section 5 gives three numerical examples to verify the accuracy and the application of FEM/Form. Finally, Section 6 draws some conclusions on our work.

2. Frictional contact

Assume that M_1 and M_2 are the points on contact body Ω_1 and Ω_2 respectively. The velocity vectors of the said points are $\dot{\mathbf{u}}_1$ and $\dot{\mathbf{u}}_2$. In the local frame, the relative velocity vector is expressed as $\dot{\mathbf{u}} = \dot{\mathbf{u}}_1 - \dot{\mathbf{u}}_2$. The relative velocity vector $\dot{\mathbf{u}}$ and the contact reaction vector $\mathbf{r}$ can be uniquely decomposed into a normal part $\dot{u}_n\mathbf{n}$, $r_n\mathbf{n}$, where $\mathbf{n}$ is the normal direction, and a tangential part $\dot{\mathbf{u}}_t$, $\mathbf{r}_t$. The contact kinematic is shown in Fig. 1. The vectors in the local contact system are:

$$\begin{cases} \dot{\mathbf{u}} = \dot{\mathbf{u}}_t + \dot{u}_n\mathbf{n} = \dot{u}_{t1}\mathbf{t}_1 + \dot{u}_{t2}\mathbf{t}_2 + \dot{u}_n\mathbf{n}, \\ \mathbf{r} = \mathbf{r}_t + r_n\mathbf{n} = r_{t1}\mathbf{t}_1 + r_{t2}\mathbf{t}_2 + r_n\mathbf{n}. \end{cases} \tag{1}$$

with

$$\mathbf{t}_1 = \{1 \quad 0 \quad 0\}^T,\ \mathbf{t}_2 = \{0 \quad 1 \quad 0\}^T,\ \mathbf{n} = \{0 \quad 0 \quad 1\}^T. \tag{2}$$

Therefore, under the local contact kinematic frame, a complete constitutive law of frictional contact, which combines the Signorini condition and Coulombs law, can be expressed in a compact form as:

$$\begin{aligned} &\text{Separation}: \mathrm{r}_n = 0,\quad \dot{u}_n \geq 0; \\ &\text{Sticking}: \quad \mathbf{r} \in \mathrm{Int}(K_\mu)\ ,\quad \dot{\mathbf{u}} = \mathbf{0}; \\ &\text{Sliding}: \quad r_n > 0\ ,\quad \mathbf{r} \in \mathrm{Bou}(K_\mu),\quad \dot{u}_n = 0,\quad \dot{\mathbf{u}}_t = -\lambda\frac{\mathbf{r}_t}{\|\mathbf{r}_t\|}. \end{aligned} \tag{3}$$

where, $\mathrm{Int}(K_\mu)$ and $\mathrm{Bou}(K_\mu)$ denote the interior and the boundary of the Coulomb cone set K_μ respectively, K_μ is defined as

$$K_\mu = \{\mathbf{r} \in \mathbb{R}^3 | \|\mathbf{r}_\mathbf{t}\| - \mu r_n \leq 0\} \tag{4}$$

The constitutive law of frictional contact is typical of implicit materials models [3]. Therefore, it is possible to apply the bi-potential theory to solve contact laws equations [5]. On the basis of de Saxcé and Feng's work, the bi-potential function of frictional contact is stated as follows:

$$b(-\dot{\mathbf{u}}, \mathbf{r}) = \Psi_{K_\mu}(\mathbf{r}) + \Psi_{\mathbb{R}_-}(-\dot{u}_n) + \mu r_n\|-\dot{\mathbf{u}}_t\|, \tag{5}$$

where $\Psi_{K_\mu}(\mathbf{r})$ represents the so-called indicator function of the closed convex set K_μ. if $\mathbf{r} \in K_\mu$, $\Psi_{K_\mu}(\mathbf{r}) = 0$; otherwise $\Psi_{K_\mu}(\mathbf{r}) = +\infty$. Consider that

$$\Psi_{K_\mu}(\mathbf{r}) + \Psi_{\mathbb{R}_-}(-\dot{u}_n) + \mu r_n\|-\dot{\mathbf{u}}_t\| \geq -(\dot{\mathbf{u}}_t\cdot\mathbf{r}_t + \dot{u}_n r_n) \tag{6}$$

On the basis of the bi-potential theory, we know that

$$\forall \mathbf{r},\ \mathbf{r}' \in K_\mu,\quad b(-\dot{\mathbf{u}}, \mathbf{r}') - b(-\dot{\mathbf{u}}, \mathbf{r}) \geq -\dot{\mathbf{u}}\cdot(\mathbf{r}' - \mathbf{r}) \tag{7}$$

Then, by applying the augmented Lagrangian method to the bi-potential function, we obtain

$$\forall \mathbf{r},\ \mathbf{r}' \in K_\mu, \rho b(-\dot{\mathbf{u}}, \mathbf{r}') - \rho b(-\dot{\mathbf{u}}, \mathbf{r}) + \{\mathbf{r} - [\mathbf{r} + \rho(-\dot{\mathbf{u}})]\}\cdot(\mathbf{r}' - \mathbf{r}) \geq 0. \tag{8}$$

In the above equation, ρ is chosen to strictly positive to ensure numerical convergence [3]. In Eq. (8), $\mathbf{r}$ is the proximal point of the augmented force $\hat{\mathbf{r}} = \mathbf{r} + \rho(-\dot{\mathbf{u}})$ with respect to $\rho b(-\dot{\mathbf{u}}, \mathbf{r})$:

$$\mathbf{r} = \mathrm{prox}[\mathbf{r} + \rho(-\dot{\mathbf{u}}), \rho b(-\dot{\mathbf{u}}, \mathbf{r})] \tag{9}$$

The inequality above can be arranged as

$$\forall \mathbf{r}' \in K_\mu, \left(\mathbf{r} - \hat{\mathbf{r}}\right)\cdot(\mathbf{r}' - \mathbf{r}) \geq 0, \tag{10}$$

where the augmented reaction force $\hat{r}$ is defined by

$$\hat{\mathbf{r}} = \mathbf{r} - \rho(\dot{\mathbf{u}}_t + (\dot{u}_n + \mu\|\dot{\mathbf{u}}_t\|)\mathbf{n}) \tag{11}$$

Eq. (10) implies that $\mathbf{r}$ is the projection of $\hat{\mathbf{r}}$ onto the closed convex Coulomb's cone.

$$\mathbf{r} = \mathrm{proj}\left(\hat{\mathbf{r}}, K_\mu\right) \tag{12}$$

The Uzawa Algorithm appears suitable to calculate the coupling variables $(-\dot{\mathbf{u}}, \mathbf{r})$ in a frictional contact situation. It consists in the association of a *predictor* and a *corrector*:

$$\begin{aligned} &\text{Predictor: } \hat{\mathbf{r}}^{(i+1)} = \mathbf{r}^{(i)} - \rho\left[\dot{\mathbf{u}}_t^{(i)} + \left(\dot{u}_n^{(i)} + \mu\left\|\dot{\mathbf{u}}_t^{(i)}\right\|\right)\mathbf{n}\right], \\ &\text{Corrector: } \mathbf{r}^{(i+1)} = \mathrm{proj}\left(\hat{\mathbf{r}}^{(i+1)}, K_\mu\right) \end{aligned} \tag{13}$$

The *corrector* integrates three conditions: *separating* ($\hat{\mathbf{r}} \in K_\mu^*$), *contact with sticking* ($\hat{\mathbf{r}} \in K_\mu$) and *contact with sliding* ($\hat{\mathbf{r}} \in \mathbb{R}^3 - (K_\mu \cup K_\mu^*)$). The corrector can be explicitly defined as follows:

$$\begin{aligned} &\text{Separation: if} \quad \mu\left\|\hat{\mathbf{r}}_t^{(i+1)}\right\| < -\hat{\mathbf{r}}_n^{(i+1)} \quad \text{then} \quad \mathbf{r}^{(i+1)} = 0; \\ &\text{Sticking:} \quad \text{else if} \left\|\hat{\mathbf{r}}_t^{(i+1)}\right\| \leq \mu\tau_n^{(i+1)} \quad \text{then} \quad \mathbf{r}^{(i+1)} = \hat{\mathbf{r}}^{(i+1)}; \\ &\text{Sliding:} \quad \text{else } \mathbf{r}^{(i+1)} = \hat{\mathbf{r}}^{(i+1)} - \frac{\left(\left\|\hat{\mathbf{r}}_t^{(i+1)}\right\| - \mu\hat{\mathbf{r}}_n^{(i+1)}\right)}{1+\mu^2}\left(\frac{\hat{\mathbf{r}}_t^{(i+1)}}{\left\|\hat{\mathbf{r}}_t^{(i+1)}\right\|} + \mu\mathbf{n}\right) \end{aligned} \tag{14}$$

3. Elasto-plasticity under large deformation

The FEM remains widely used to simulate the behavior of elasto-plastic materials, and the RMA is widely used when it comes to dealing