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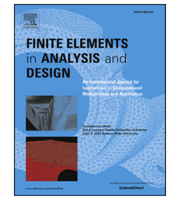

A solid-shell Cosserat point element for the analysis of geometrically linear and nonlinear laminated composite structures

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ABSTRACT

This study deals with the development of a new solid-shell element using the Cosserat point theory for the linear and nonlinear analysis of laminated elastic structures. Generally speaking, the Cosserat point approach considers the element as a structure with a strain energy function that characterizes its response. This strain energy function is additively decomposed into two parts, where the first part depends on an average measure of the deformation and the second part, which is referred to as the inhomogeneous strain energy, controls the element's response to any inhomogeneous deformation. Due to the coupling nature between homogeneous and inhomogeneous deformation in laminated structures, the inhomogeneous strain energy is further additively decomposed into two parts. The first part quadratically depends on the inhomogeneous strain measures, while the second part accounts for the coupling between the homogeneous and inhomogeneous deformations. In the present study, a methodology for the determination of the constitutive coefficients for the two parts of the inhomogeneous strain energy function is presented. The resulting constitutive coefficients ensure an accurate modeling of the inhomogeneous deformations and also ensure that the element has a control on all the inhomogeneous modes of the deformation. Both linear and nonlinear example problems are considered, which demonstrate that the developed laminated Cosserat point element (LSSCPE) is accurate, efficient, robust, and applicable in modeling laminated structures with one element through the structure's thickness.

1. Introduction

Laminated composite materials have been increasingly applied in various engineering sectors (e.g. aerospace, auto-mobile, submarine, and biomedical engineering) for constructing lightweight structures [1,2]. Thus, various plate and shell theories have been formulated to predict the response of laminated composite plates and shells [3–8] among other, and numerical tools have been developed to analyse the response of laminated structures with complex geometry. The most common numerical tool for this purpose is the finite element method. In particular, finite elements that are designed to model thin structures via one element through the structure's thickness can be broadly classified into the following three categories: (i) shell elements based on various laminated plate/shell theories; (ii) degenerated shell elements directly obtained from a fully three-dimensional continuum theory; and (iii) solid-shell elements derived from three-dimensional solid elements.

Elements based on laminated plate/shell theories can be subdivided into three groups, namely, elements based on the equivalent single layer (ESL) theories, elements based on layerwise models, and elements based on Zig-Zag theory [9–12]. Elements based on single layer theories are suitable for modeling the global response of laminated structures and they are computationally efficient as the number of unknowns is independent of the number of layers constituting the laminate [13–22]. In contrast, element formulations based on layerwise theories [23–28] are designed to predict the local response of the laminate by assuming independent displacement fields for each layer, and can be applied for regions where the stress distribution is of primary interest. However, this comes at the expense of a higher computational cost as the number of unknown parameters depends on the number of layers. Attempts to combine the advantages of ESL and layerwise theories have led to the development of the Zig-Zag models [29–35]. Furthermore, these attempts have led to the development of the multiscale laminated plate theory [36] and the global-local plate and shell elements [37–39].

As in shell elements, the degenerated shell elements [40] model a shell in terms of mid-surface nodal variable consisting of both translation and rotational variables. Numerous successful degenerated shell elements were developed for the analysis of laminated composite structures [41–47] among others. As a result of the involvement of rota-

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tional degrees-of-freedom, both shell and degenerated shell elements suffer from several disadvantages, such as: the required special transition elements when solid elements are combined with such elements, and the required special rotation interpolation strategies for preserving the objectivity of strain measures with respect to a rigid motion [48,49].

On the other hand, solid-shell elements possess only displacement degrees of freedom that are located at the bottom and top surfaces. Therefore, the aforementioned drawbacks, which are associated with both shell and degenerated shell elements, are naturally avoided when using solid-shell elements. In addition, the implementation of general three-dimensional constitutive equations into solid-shell elements and the exact evaluation of stress distribution along the thickness are straightforward. As a result, the development and improvement of solid-shell elements have recently gained considerable interest and different well-established techniques, such as: assumed natural strain (ANS), enhanced assumed strain (EAS), and hybrid stress methods that must be applied for treating different locking phenomena including volumetric locking, membrane locking, Poisson-thickness locking, transverse shear locking, and curvature-thickness (trapezoidal) locking [50–64]. However, the first contribution with regard to the analysis of laminated structures via one solid-shell element through the laminate thickness is the work of [65]. Specifically [65], have used the ANS and EAS methods to construct their element formulation. Using only one element through the laminate thickness, the number of degrees of freedom was independent of the layer number. Since then, several advanced solid-shell elements for modeling laminated structures have been developed [66–77].

Recently, a novel finite element technology based on the Cosserat point theory [78–82] was developed and applied in the formulation of a 3D brick element for the numerical solution of problems for nonlinear hyperelastic materials. Generally speaking, the Cosserat point element considers an element as a structure and introduces a strain energy function that characterizes its response to homogeneous and inhomogeneous deformations. Once the strain energy of the Cosserat point element has been specified, integration over the element region is not required and the response of the element is hyperelastic. From the numerical point of view, it was observed that the 3-D brick Cosserat point element is a robust and accurate element that can be used to characterize the response of thin plates and shells with only one element through the thickness, as well as, complicated three-dimensional structures [79–82]. With regards to solid-shell developments, [64] have developed a solid-shell element using the Cosserat point theory and showed the applicability of the developed element in modeling thin structures. Specifically, [64] modified the volume average of the Green-Lagrange strain tensor and introduced the assumed natural inhomogeneous strain concept in order to eliminate the well-known transverse shear locking and curvature-thickness locking in solid-shell elements.

In the present study, a solid-shell finite element formulation, which is based on the Cosserat point theory, is developed for the linear and nonlinear analysis of laminated structures. This element will be referred to as a laminated solid-shell Cosserat point element (LSSCPE). As it is mentioned before, the response of the Cosserat point element is determined by proposing a strain energy function. Generally speaking, this strain energy function is additively decomposed into a homogeneous part and an inhomogeneous part, where the latter part is assumed to quadratically depend on the inhomogeneous strain measures. For laminated composite structures, the quadratic form of the inhomogenous strain energy function is insufficient due to the coupling between homogeneous strains and inhomogeneous strain measures. Therefore, in the present study, the inhomogeneous strain energy function is enriched by adding additional term that controls the coupling between the homogeneous deformations and inhomogeneous ones. The two terms constituting the inhomogeneous strain energy function depend on sets of constitutive coefficients, which control the accuracy and the stability of the element. In the present study, a methodology for the determination of the constitutive coefficients is introduced. These constitutive coefficients are determined by integrating a three-dimensional strain energy function that quadratically depends on a strain field, where the latter is additively decomposed into a modified compatible part and an enhanced part. The compatible strain field is modified based on the assumed natural inhomogeneous strain concept proposed in Ref. [64] to avoid both transverse shear locking and curvature-thickness locking. The enhanced part of the strain field is designed to avoid Poisson-thickness locking, volumetric-locking, as well as, to improve the accuracy for bending. The resulting constitutive coefficients lead to a solid-shell element, which is accurate, robust, totally free of locking and insensitive (as much as possible) to mesh distortion. From the computational point of view, the developed laminated solid-shell Cosserat point element is efficient, since no integration is required to calculate both the internal nodal forces vector and the tangent stiffness matrix.

The paper is outlined as follows: Section 2 presents the basic kinematic quantities of the laminated solid-shell Cosserat point element (LSSCPE), the assumed natural inhomogeneous strains (ANIS) concept, and the strain energy function of the LSSCPE. Section 3 describes the procedure for determining the constitutive coefficients of the inhomogeneous strain energy function. In Section 4, the nodal internal forces and the tangent stiffness matrix for the finite element formulation are developed. Section 5 presents a pseudo code for the implementation of the LSSCPE and Section 6 presents a numerical study of the performance of the developed LSSCPE. Finally, Section 7 presents the conclusions.

2. Theoretical background

In this section, the basic kinematic quantities of the laminated solid-shell Cosserat point element (LSSCPE), the assumed natural inhomogeneous strain (ANIS) concept, and the strain energy function of the LSSCPE will be presented.

2.1. Kinematics

Let $\left\{\overline{\mathbf{D}}_i,\ \overline{\mathbf{d}}_i\right\}$ be the location of the eight nodes of the finite element in the reference and deformed configurations, respectively (see Fig. 1). Using the Bubnov-Galerkin approach the location of a material point in the reference configuration, $\mathbf{X}^*$, and the location of the same material point in the deformed configuration, $\mathbf{x}^*$, are given by

$$\mathbf{X}^* = \sum_{i=0}^{7} \overline{N}^i\left(\theta^1,\ \theta^2,\ \theta^3\right)\overline{\mathbf{D}}_i, \quad \mathbf{x}^* = \sum_{i=0}^{7} \overline{N}^i\left(\theta^1,\ \theta^2,\ \theta^3\right)\overline{\mathbf{d}}_i, \tag{1}$$

where, $\overline{N}^i$, are the standard tri-linear shape functions, which satisfy the Kronecker property, and $-1/2 \leqslant \theta^i \leqslant 1/2,\ (i = 1, 2, 3)$ are the convected coordinates. For the kinematic description, it is convenient to express the position vectors $\{\mathbf{X}^*,\ \mathbf{x}^*\}$ in terms of generalized variables, which will be referred to as element directors of both reference and deformed configurations, as follows

$$\mathbf{X}^* = \sum_{i=0}^{7} N^i\left(\theta^1,\ \theta^2,\ \theta^3\right)\mathbf{D}_i, \quad \mathbf{x}^* = \sum_{i=0}^{7} N^i\left(\theta^1,\ \theta^2,\ \theta^3\right)\mathbf{d}_i, \tag{2}$$

where N^i are the tri-linear shape functions defined by

$$\begin{aligned} &N^0 = 1, \quad N^1 = \theta^1, \quad N^2 = \theta^2, \quad N^3 = \theta^3, \\ &N^4 = \theta^1\theta^2, \quad N^5 = \theta^1\theta^3, \quad N^6 = \theta^2\theta^3, \quad N^7 = \theta^1\theta^2\theta^3. \end{aligned} \tag{3}$$