Finite Elements in Analysis and Design 141 (2018) 1–16

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Electromagnetic wave propagation analysis by an explicit adaptive technique based on connected space-time discretizations

Delfim Soares Jr. *, Danielle R. de M. Leal

Faculty of Engineering, Federal University of Juiz de Fora, CEP 36036-330, Juiz de Fora, MG, Brazil

ARTICLE INFO

Keywords:
Wave propagation
Maxwell equations
Adaptive parameters
Time domain analysis
Explicit techniques
Subcycling

ABSTRACT

In this work, a new explicit time marching technique is considered to analyse 2D electromagnetic wave propagation problems. The technique considers adaptive time integrators, which are spatially and temporally locally computed, providing a connection between the adopted spatial and temporal discretizations. This approach allows the errors produced by both discretization procedures to be counterbalanced, enabling more accurate analyses. A multi time-steps methodology is also considered here, associated to subcycling techniques, enhancing the efficiency and the adaptive features of the method. As it is described along the paper, the new technique is very effective, robust and simple to implement, providing a very suitable numerical approach to analyse complex wave propagation models.

1. Introduction

Electromagnetic wave propagation phenomena have numerous applications in various branches of science and in practical engineering design. Since it is usually very difficult to obtain analytical transient responses for these models, numerical techniques must be applied to find approximate solutions, and step-by-step time integration algorithms are routinely employed when a detailed description of the wave propagation evolution is required.

The literature reports many classical explicit [1–5] and implicit [6–10] algorithms for time-marching analysis (for a comprehensive review, see Ref. [11]). Explicit procedures are usually preferable because of their lower computational effort; however, there are restrictions in their use due to stability conditions. Implicit approaches, on the other hand, are generally unconditionally stable; however, they are usually characterized by higher computational costs. Many procedures can be employed to improve the stability and accuracy of time integration algorithms, such as subcycling techniques [12–14], high-order accurate schemes [15–20], automatic time step control [21–25], etc. As a matter of fact, a lot of research is continuously realized on this field and several time marching techniques are available nowadays for wave propagation analysis [26–39].

Numerous efforts during the past several decades have focused on developing time integration algorithms that include controllable numerical dissipation in the high-frequency response domain. The purpose of this numerical dissipation is to reduce the spurious, non-physical oscillations that sometimes occur due to the excitation of spatially unresolved modes. One basic difficulty in designing such algorithms is to add high-frequency dissipation without introducing excessive algorithmic damping in the important low-frequency modes. Considering time marching techniques, numerical dissipation can be measured by the spectral radius of the method, which is defined as the largest magnitude of the eigenvalues of the numerical amplification matrix. In a stable analysis, the spectral radius varies between zero and one, and when the spectral radius is unitary, the technique is non-dissipative. Numerical damping is introduced when the spectral radius becomes lower than one. In the extreme case, the spectral radius can be reduced to zero (or close to zero), providing maximal dissipation and allowing specific frequency responses to be very quickly eliminated. Several implicit time-marching techniques with dissipative properties are available nowadays for hyperbolic models [6–10,29–35]; however, as described by Hulbert and Chung [1], numerical dissipation is also important (and perhaps more important) when solving wave propagation problems using explicit methods, and few works focus on this topic [40–43]. The principal use of explicit time integration methods is for problems in which the time step size needed for accuracy is of the same order as the step size limit dictated by the stability limit of an explicit method (e.g., wave propagation and impact problems). The response of these problems is usually characterized by large gradients and/or discontinuities in the solution due to propagating wave fronts. Among explicit time integration methods, the

* Corresponding author.
E-mail address: delfim.soares@ufjf.edu.br (D. Soares).

https://doi.org/10.1016/j.finel.2017.11.002
Received 2 June 2017; Received in revised form 13 November 2017; Accepted 13 November 2017

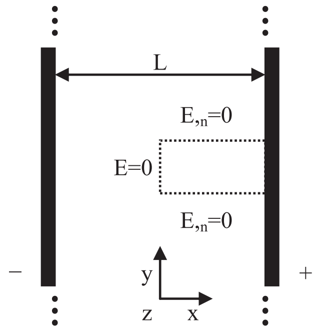

Fig. 1. Sketch of the first model: regions of electric currents (parallel lines) and discretized domain.

nearly universal choice is the Central Difference Method (CDM), which possesses no numerical dissipation. Thus, spurious oscillations may occur in the solutions computed using the CDM.

In this work, a new explicit time marching procedure is proposed to analyse electromagnetic wave propagation models, in which enhanced dissipation control is enabled. Here, the spatial discretization of the model is carried out by employing the Finite Element Method (FEM) [44–46] and an adaptive time marching technique is implemented, which considers a connection between the adopted temporal and spatial discretizations. Thus, in this methodology, an adaptive time integrator is considered, which assumes different values for each finite element and for each time step, and these values are computed taking into account the physical/geometrical properties of the elements of the spatial discretization, the adopted time-step, and local previous time step results. The evaluation of this parameter focuses on enabling an effective numerical dissipative algorithm, aiming to eliminate the influence of spurious modes and to reduce amplitude decay errors. It defines the so-called dissipative and non-dissipative elements of the model, which are relabeled at each time step of the analysis. The proposed adaptive strategy is non-iterative and only based on single-step relations involving two variables: the unknown field itself and its first time derivative. Thus, just a trivial single set of equations has to be dealt with within a time-step (explicit analysis), and the resulting method stands as truly self-starting, eliminating any kind of cumbersome initial procedure, such as the computation of initial second time derivative values and/or the computation of multistep initial values.

The basic framework of the recursive time-marching relations discussed here was presented by Soares [39], taking into account non-adaptive procedures. Later on, this initial work was extended for implicit formulations, considering adaptive approaches [35]. Explicit algorithms were also developed, considering non-transient adaptive procedures [42]; in this case, the time integration parameters were just adapted once, according to the features of the FEM matrices. Here, a fully explicit adaptive formulation is presented, in which the time integration parameter not only adapts itself according to the properties of the model and the adopted discretizations, but also as the solution evolves. In addition, the time discretization itself is also locally adapted, according to the adopted spatial discretization (subcycling).

The manuscript is organized as follows: first (section 2), the governing equations of the model are briefly presented and, in the sequence, the adopted spatial and temporal numerical discretizations are discussed (section 3), describing the proposed adaptive technique. In section 4, numerical results are considered, illustrating the accuracy and effectiveness of the methodology. Further details on the mathematical formulation of the new approach are provided in the appendix, where the stability and dissipative features of the technique are more deeply discussed.

2. Governing equations

Maxwell's equations in differential form can be written as follows:

$$e_{ijk}E_{k,j} = -\dot{B}_i \tag{1a}$$

$$e_{ijk}H_{k,j} = \dot{D}_i + J_i \tag{1b}$$

$$D_{i,i} = \rho \tag{1c}$$

$$B_{i,i} = 0 \tag{1d}$$

where indicial notation for Cartesian axes is considered and e_{ijk} stands for the permutation symbol (also known as alternator tensor). Subscript commas and overdots indicate partial space and time derivatives, respectively (i.e., $V_{i,j} = \partial V_i/\partial x_j$ and $\dot{V}_i = \partial V_i/\partial t$, where $V_i(X,t)$ stands for a generic vector field representation and X and t denote its spatial and temporal arguments, respectively).

In equation (1), E_i and H_i are the electric and magnetic field intensity components, respectively; D_i and B_i represent the electric and magnetic flux density, respectively; and J_i and ρ stand for the electric current and electric charge density, respectively. The constitutive relations between the field quantities are specified as follows:

$$D_i = \varepsilon\, E_i \tag{2a}$$

$$B_i = \mu\, H_i \tag{2b}$$

$$J_i = \sigma\, E_i \tag{2c}$$

where the parameters ε, μ and σ denote, respectively, the permittivity,

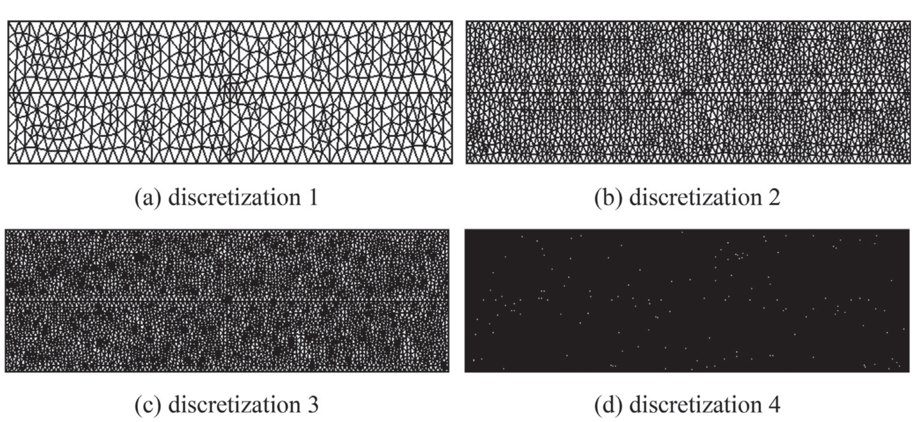

Fig. 2. Adopted discretizations for the first model: (a) 1008 elements and $\Delta t_0 = 2.1965{\cdot}10^{-11}s$; (b) 3458 elements and $\Delta t_0 = 1.1728{\cdot}10^{-11}s$; (c) 13906 elements and $\Delta t_0 = 5.7305{\cdot}10^{-12}s$; and (d) 33472 elements and $\Delta t_0 = 3.1282{\cdot}10^{-12}s$.