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Five node pyramid elements for explicit time integration in nonlinear solid dynamics

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A B S T R A C T

Pyramid finite elements have become increasingly popular to facilitate meshing due to their distinctive shape and innate ability to transition naturally between elements with quadrilateral and triangular faces. This is especially important for explicit time integration, as the alternative of constraints can be problematic for wave propagation applications frequently modeled by such methods. Although formulations to alleviate singularity problems near the apex node have existed for decades, pyramid elements are not available in typical explicit solid dynamics software. Several 5-node pyramid approaches are evaluated herein for suitability as transition elements in lumped mass explicit methods for nonlinear solid dynamics. Several typical pyramid elements generated from both hexahedral shape functions and with Bedrosian rabbit functions are extended to explicit temporal methods by the development of mass lumping and critical time increment estimation schemes. Standard and uniform strain hexahedrons are also degenerated into a pyramid by simple nodal duplication in the connectivity. A focus of the study is on the viability of using only existing hexahedron capabilities typically available in many explicit codes. Performance is assessed in standard benchmark problems and practical applications using various elastic and elastic-plastic material models and involving large strains/deformations, severe distortion, and contact-impact. Examples first evaluate the elements on their own and then for the principal case as transitions within a hexahedral-dominant model. Row-summation mass lumping is shown to be the best method for any pyramid element approach, which may require slight coding changes for degenerated hexahedrons. The results indicate that it may also be a good systematic method for mass lumping of general degenerate hexahedral types. The single quadrature point standard and Bedrosian pyramid elements are also found to be robust and the best performers, particularly requiring significantly fewer computations than the degenerated uniform strain hexahedron. If used properly, however, all element types are demonstrated to perform satisfactorily (and identically) and thus demonstrate their viability and benefits for practical applications using hexahedral-dominant meshing.

1. Introduction and background

Pyramid finite elements are not typically used alone in models, but are useful for modeling transition regions in hexahedral-dominant meshes. The first-order 5-node pyramid elements, considered herein, have a quadrilateral base face and four triangular faces (see Fig. 1). Their mixture of both quadrilateral and triangular faces naturally facilitates systematic transition between hexahedral and tetrahedral regions without the use of multipoint constraints. Wave propagation is frequently an important component of explicit analyses and constraints, such as used for mesh tying of multiple regions, can introduce significant wave dispersion errors at tied interfaces. This is an important contrast to static or slower implicit analyses where mesh tying can be quite effective. Analysts historically have created such “Hex-Dominant” models directly or may use more recent automatic meshing approaches, e.g., [1–3], that generate only brick elements until they encounter difficult regions that they then fill with combinations of wedge, tetrahedral, and pyramid elements.

Although wedge elements are also suitable for such transitions, the geometries of pyramid and wedge elements provide their own distinct meshing advantages and limitations. Wedge-hexahedral meshes are naturally created by extruding/sweeping two-dimensional triangle-quadrilateral meshes. Pyramids can facilitate more complex three-dimensional connections between hexahedrons and tetrahedrons and/or wedges. Unstructured tetrahedral meshing approaches can also easily grid complicated geometries, but they typically are considerably more

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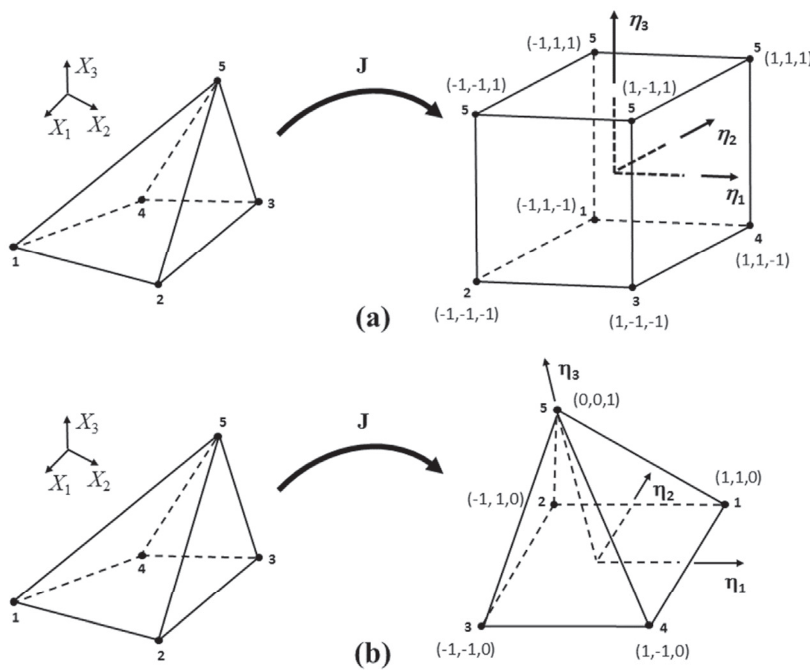

Fig. 1. Five node type pyramid finite elements (PYR5) configuration (left) showing two mapping options into a parent element on the right (a) mapped into a hexahedron and (b) mapped into a pyramid.

computationally expensive, since they generally consist of many more elements (usually by one or more orders of magnitude). Automated all-Hex meshing also continues to improve in quality and model size, but each of these approaches still have their own pros and cons. Hex-Dominant approaches are available in various popular meshing software, e.g., ABAQUS CAE [3], and offer a compromise by producing models that predominately consist of the preferred hex elements while providing the desirable more automatic characteristic of a "tet" mesher. Instead of automatically meshing with tests for the entire mesh, the automation is applied only to regions where the "hex" mesher has trouble. Despite the accuracy sacrificed in certain regions (transition and non-hexahedron), the reduced meshing time for the analyst may be well worthwhile and the solution time typically is significantly less than with an all tetrahedral mesh.

Unfortunately, pyramids generally do not perform as well as brick elements, especially first-order ones that typically can be overly stiff in flexure or in nearly incompressible material applications. They preferably should be well-shaped and used for noncritical regions of low stress/strain gradients. Pyramid elements are also prone to singularity issues near the apex node, but Bedrosian [4] demonstrated the significant benefits of using "rabbit" functions over traditional finite element functions, e.g., a degenerated hexahedron, to overcome these difficulties in displacement-based formulations. The standard pyramid exhibits these singularities at the apex [4], since it is developed by collapsing a face of a hexahedron into a single node and maps the pyramid into the parent isoparametric hexahedron, as shown Fig. 1(a). Standard Gauss-Legendre quadrature for the hexahedron may thus be used for this element, but a large number of quadrature points may be necessary and a strongly skewed pyramid may still be problematic. The Bedrosian element [4] maps into a parent pyramid element, as shown Fig. 1(b), and the rabbit function pyramids also contain singularities near the apex, but much weaker ones that provide accurate results in C^0 continuity formulations with low-order numerical quadrature. Felippa [5] later developed special quadrature rules, which recognize that the mapping is from a pyramidal shape, that are also accurate with low-order numerical quadrature. Despite these findings, pyramid elements have not generally appeared in popular nonlinear solid mechanics finite element codes, particularly not in ones using explicit integration for high rate dynamics, which historically have contained mostly first-order C^0 continuity elements. Classical finite element analysis (FEA) continues to be a primary computational method of choice for most solid mechanics applications and the explicit method is significantly used by analysts in many industries such as defense, crashworthiness, and metal forming. The explicit lumped mass approach, without the use of a stiffness matrix, uses many small time increments with a central difference scheme to march through time. It is thus well suited for rapidly changing/high-rate short duration applications, but can produce distinct nuances and severely affect element performances differently than in typical static/implicit methods [6]. Several additional features must also be developed for the lumped mass explicit method that are not needed with static/implicit methods.

Although second-order elements have recently emerged in explicit codes [6–9] and show benefits for Hex-Dominant modeling in many cases [8], first-order elements have long dominated explicit analyses and may still be the better choice for many modeling situations. In static/implicit analyses where additional element costs may be small compared to the equation solving, it may be desirable to avoid potential problems by splitting pyramids into tetrahedra, but the cost of an explicit analysis is directly proportional to the number of elements, and element splits can drive down the critical time increment to increase CPU time. It thus seems to be important to include first-order pyramid elements and/or pyramid element capabilities, e.g., degenerate hexahedrons, as options in explicit analysis codes to be able to better exploit hexahedral dominant meshing for general modeling. In Ref. [10], the uniform strain hexahedron of Flanagan-Belytschko [11] was degenerated into a variety of transition elements for Hex-Dominant explicit analyses, including a pyramid, by various simple duplications of nodal definitions in the connectivity. Although the results of this initial cursory investigation are promising, they did not rigorously look at mass lumping, hourglass modes, locking, or degenerate surfaces. Since the Flanagan-Belytschko formulation uses closed form expressions to compute the volume and uniform gradients, it may naturally avoid the singularities with the degenerated standard hexahedral functions near the pyramid apex node that can be problematic with numerical integration. The Flanagan-Belytschko hexahedron is commonly contained in many