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Adaptive time stepping for fluid-structure interaction solvers

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ABSTRACT

A novel adaptive time stepping scheme for fluid-structure interaction (FSI) problems is proposed that allows for controlling the accuracy of the time-discrete solution. Furthermore, it eases practical computations by providing an efficient and very robust time step size selection. This has proven to be very useful, especially when addressing new physical problems, where no educated guess for an appropriate time step size is available. The fluid and the structure field, but also the fluid-structure interface are taken into account for the purpose of *a posteriori* error estimation, rendering it easy to implement and only adding negligible additional cost. The adaptive time stepping scheme is incorporated into a monolithic solution framework, but can straightforwardly be applied to partitioned solvers as well. The basic idea can be extended to the coupling of an arbitrary number of physical models. Accuracy and efficiency of the proposed method are studied in a variety of numerical examples ranging from academic benchmark tests to complex biomedical applications like the pulsatile blood flow through an abdominal aortic aneurysm. The demonstrated accuracy of the time-discrete solution in combination with reduced computational cost make this algorithm very appealing in all kinds of FSI applications.

1. Introduction

Many scientific and engineering problems involve the coupling of several physical effects or models. One class of coupled problems is the interaction of fluid flow and solid bodies. Possible applications range from aero-elasticity over civil engineering to biomedical problems like the analysis of blood flow in the human vascular system. Application-wise and from a numerical point of view, the interaction of an incompressible fluid flow with solid bodies undergoing finite deformation is of particular interest. Although many researchers addressed this class of problems for decades, solving *fluid-structure interaction (FSI)* problems numerically still poses a challenging task.

Most problems in FSI are transient and, thus, involve temporal discretization and time integration. To allow for the computation of a temporally accurate solution while simultaneously limiting the computational cost, we propose an adaptive time stepping algorithm for FSI problems based on *a posteriori* error estimation. To the authors' best knowledge, such an approach is not described in literature yet, and we aim at closing this gap. In our FSI solver, we allow for the possibility of choosing time integration schemes for the solid and the fluid field independently and tailored to each field's needs as recently introduced by Mayr et al. [1] in the context of monolithic FSI solvers, but require the time step size to be the same in both fields. The error of the fully implicit marching time integration scheme is estimated with the help of an auxiliary explicit scheme. Due to the explicit character of the auxiliary scheme, computational cost associated with the error estimation is negligible. As FSI is a surface-coupled problem, the fluid field and the structure field, but also the fluid-structure interface are taken into account for error estimation. The estimated error is then used to adapt the time step size throughout the entire simulation such that the error does not exceed a user-given tolerance. The algorithm ensures that the time step size is chosen such that every portion of the FSI domain satisfies its individual demand for accuracy. At the same time, huge savings of computational cost can be achieved w.r.t. two aspects. Firstly, wall clock time of a single run of a simulation can be significantly reduced compared to the case of non-adaptive time stepping. Secondly, finding a suitable time step size via an adaptive procedure is beneficial when one needs to simulate problems where no educated guess for an appropriate time step size is known. The adaptive scheme chooses a suitable time step size by itself and prevents the need of several trial runs with differ-

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ent values for the pure purpose of finding an appropriate value for the time step size. To the authors' best knowledge, this is the first adaptive time stepping algorithm for nonlinear FSI problems undergoing large deformation.

In this contribution, we use the proposed adaptive time stepping scheme to study the interaction of an incompressible fluid flow in ALE description with a deformable solid body. We solve the problem with a fully implicit finite-element-based monolithic FSI solver as previously described in Refs. [1,2], but we will also address the algorithmic layout in case of partitioned solution schemes. We base our approach on adaptive time stepping schemes for single-field problems that are well-known for decades, e.g. for solid dynamics [3–13] and fluid dynamics applications [14–21].

Optimal design of time integration schemes is essential to guarantee stability and accuracy in FSI computations. Many efforts have been undertaken to develop stable time integration routines for ALE-based fluid computations on moving domains [22–26]. The possibility of choosing time integration schemes for the solid and the fluid field independently and tailored to each field's needs has recently been introduced by Mayr et al. [1] in the context of monolithic FSI solvers.

Adaptive time stepping schemes for the solution of *ordinary differential equations (ODEs)* date back roughly a century. Based on an approximation of the error of the time-discrete solution, they adapt the time step size Δt_n to match a user-given level of accuracy. Schemes relying on *a posteriori* error estimation can be found in textbook literature [27–30]. Alternative approaches based on control theory have been developed by Gustafsson et al. [31] and Söderlind [32,33]. Such approaches are said to increase stability and to produce a smoother evolution of time step sizes. A detailed analysis of the analogy of these approaches to the more classical methods based on *a posteriori* error estimation is carried out in Ref. [28], for example. In this work, algorithms based on *a posteriori* error estimation produced satisfying results for FSI problems. Hence, approaches based on control theory are not considered in this manuscript, but can straightforwardly be incorporated into the presented algorithmic framework.

Recently, approaches for *goal-oriented error estimation* became very popular, where one aims at controlling the error in a user-chosen *quantity of interest*. Since these techniques usually require the solution of an adjoint problem, they become computationally and storage-wise very expensive in transient problems and, thus, are not considered in this work. However, some approaches addressing these issues are available in literature. For instance, Cyr et al. [34] use data compression techniques to reduce the huge storage demands, while Carey et al. [35] apply a block-wise adaptivity approach based on coarse scale adjoint solutions. Promising work based on modal analysis has been done by Verdugo et al. [36,37] for time-dependent solid mechanics problems.

The remainder of this manuscript is organized as follows: First, we briefly summarize the FSI problem and outline the monolithic solution procedure in Section 2. Some fundamentals of adaptive time stepping based on *a posteriori* error estimation are recalled in Section 3, before the adaptive time stepping procedure for FSI solvers is proposed in Section 4 including the discussion of important practical aspects. Section 5 presents numerical examples, that demonstrate and discuss features and properties of the proposed adaptive time stepping scheme. Finally, a summary is given in Section 6.

2. Fluid-structure interaction in a Nutshell

In the present contribution, we exemplarily study the interaction of an incompressible fluid flow with solid bodies undergoing finite deformation. We apply a monolithic solution scheme. A brief introduction to such FSI problems is given here, while a detailed description of the model, its discretization, and a thorough derivation of the monolithic solution method have been presented in Mayr et al. [1].

2.1. Physical model

We couple two physical domains, namely a deformable fluid domain $\Omega^{\mathcal{F}}$ and a solid domain $\Omega^{\mathcal{S}}$, cf. Fig. 1. To account for the moving fluid domain, an *arbitrary Lagrangean–Eulerian (ALE)* observer is used for the fluid field, while the solid body is described in a purely Lagrangean fashion. The fluid field is governed by the incompressible Navier–Stokes equations

$$\rho^{\mathcal{F}} \frac{\partial \underline{\mathbf{u}}^{\mathcal{F}}}{\partial t} + \rho^{\mathcal{F}} \left(\underline{\mathbf{u}}^{\mathcal{F}} - \underline{\mathbf{u}}^{\mathcal{G}} \right) \cdot \nabla \underline{\mathbf{u}}^{\mathcal{F}} - 2\mu^{\mathcal{F}}_{\text{dyn}} \nabla \cdot \underline{\underline{\boldsymbol{\varepsilon}}} \left(\underline{\mathbf{u}}^{\mathcal{F}} \right) + \nabla p^{\mathcal{F}} = \rho^{\mathcal{F}} \underline{\mathbf{b}}^{\mathcal{F}},$$

$$\nabla \cdot \underline{\mathbf{u}}^{\mathcal{F}} = 0,$$

with the primary unknowns $\underline{\mathbf{u}}^{\mathcal{F}}$ and $p^{\mathcal{F}}$ being the fluid velocity and pressure field, respectively. The fluid density and dynamic viscosity are denoted by $\rho^{\mathcal{F}}$ and $\mu^{\mathcal{F}}_{\text{dyn}}$, respectively, while the strain rate tensor is computed as the symmetric gradient of the fluid velocity $\underline{\mathbf{u}}^{\mathcal{F}}$. Possible body forces in the fluid field are denoted by $\underline{\mathbf{b}}^{\mathcal{F}}$. As the fluid field is described in an ALE fashion, the grid velocity $\underline{\mathbf{u}}^{\mathcal{G}}$ needs to be computed from the grid displacement field $\underline{\mathbf{d}}^{\mathcal{G}}$. For moderately deforming fluid domains, the grid displacement field $\underline{\mathbf{d}}^{\mathcal{G}}$ is determined by harmonic extension whereas large deformations require the assumption that the ALE field behaves like a pseudo-elastic solid. The solid body with density $\rho^{\mathcal{S}}$ and body force $\underline{\mathbf{b}}^{\mathcal{S}}_0$ per undeformed unit volume is governed by the nonlinear elastodynamics equation

$$\rho^{\mathcal{S}}_0 \frac{\mathrm{d}^2 \underline{\mathbf{d}}^{\mathcal{S}}}{\mathrm{d}t^2} = \nabla_0 \cdot \left(\underline{\underline{\mathbf{F}}}\, \underline{\underline{\mathbf{S}}} \right) + \underline{\mathbf{b}}^{\mathcal{S}}_0,$$

where the displacement field $\underline{\mathbf{d}}^{\mathcal{S}}$ is the primary unknown. The deformation is fully characterized by the deformation gradient $\underline{\underline{\mathbf{F}}} = \nabla_0 \underline{\mathbf{x}}^{\mathcal{S}}$. For the compressible or nearly incompressible solid, we assume a hyperelastic strain energy function $\Psi^{\mathcal{S}}$ to compute the 2nd Piola–Kirchhoff stresses $\underline{\underline{\mathbf{S}}} = 2\partial \Psi^{\mathcal{S}} / \partial \underline{\underline{\mathbf{C}}}$ using the right Cauchy–Green tensor $\underline{\underline{\mathbf{C}}} = \underline{\underline{\mathbf{F}}}^{\mathrm{T}} \underline{\underline{\mathbf{F}}}$. At the fluid-structure interface Γ_{FSI}, we require kinematic continuity of fluid and solid velocity fields, i.e. $\underline{\mathbf{u}}^{\mathcal{F}}_{\Gamma_{\text{FSI}}} \left(\underline{\mathbf{x}}, t \right) = \partial \underline{\mathbf{d}}^{\mathcal{S}}_{\Gamma_{\text{FSI}}} / \partial t \left(\underline{\mathbf{X}}, t \right)$, as well as the equilibrium of interface traction fields $\underline{\mathbf{h}}^{\mathcal{F}}_{\Gamma_{\text{FSI}}}$ and $\underline{\mathbf{h}}^{\mathcal{S}}_{\Gamma_{\text{FSI}}}$. The kinematic constraint is enforced weakly via a Lagrange multiplier field $\underline{\boldsymbol{\lambda}}$, which allows for an interpretation of the Lagrange multiplier field as the interface traction. Here, we make the arbitrary choice $\underline{\boldsymbol{\lambda}} = \underline{\mathbf{h}}^{\mathcal{S}}_{\Gamma_{\text{FSI}}} = -\underline{\mathbf{h}}^{\mathcal{F}}_{\Gamma_{\text{FSI}}}$, i.e. the Lagrange multiplier field is seen as the interface traction acting onto the solid side of the interface.

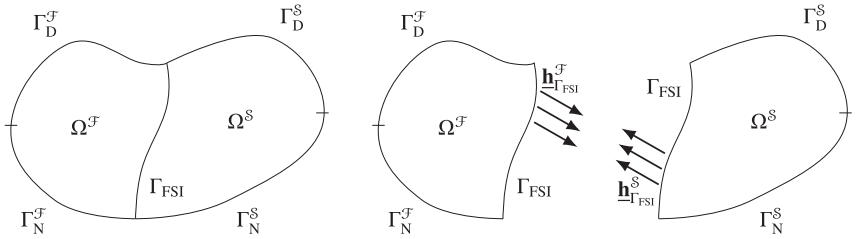

Fig. 1. Problem statement adopted from Ref. [38] — *Left:* The domain Ω is subdivided into a fluid domain $\Omega^{\mathcal{F}}$ and a structural domain $\Omega^{\mathcal{S}}$ by the fluid-structure interface Γ_{FSI}. Both subdomains are bounded by Dirichlet boundaries $\Gamma^{\mathcal{F}}_{\mathrm{D}}$ and $\Gamma^{\mathcal{S}}_{\mathrm{D}}$, Neumann boundaries $\Gamma^{\mathcal{F}}_{\mathrm{D}}$ and $\Gamma^{\mathcal{S}}_{\mathrm{D}}$, and the common fluid-structure interface Γ_{FSI}, respectively. *Right:* At the interface, kinematic continuity as well as equilibrium of interface traction fields $\underline{\mathbf{h}}^{\mathcal{F}}_{\Gamma_{\text{FSI}}}$ and $\underline{\mathbf{h}}^{\mathcal{S}}_{\Gamma_{\text{FSI}}}$ are required.