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Topology optimization with a time-integral cost functional

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ABSTRACT

We present a topology optimization based procedure aiming at the optimal placement (and design) of the supports in problems characterized by a time dependent construction process. More precisely, we focus on the solution of a time-dependent minimal compliance problem based on the classical *Solid Isotropic Material with Penalization* (SIMP) method. In particular, a continuous optimization problem with the state equation defined as the time-integral of a linear elasticity problem on a space-time domain is firstly introduced and the mean compliance over a time interval objective functional is then selected as objective function. The optimality conditions are derived and a fixed-point algorithm is introduced for the iterative computation of the optimal solution. Numerical examples showing the differences between a standard SIMP method, which only optimizes the shape at the final time, and the proposed time-dependent approach are presented and discussed.

1. Introduction

Topology optimization is a powerful design tool that is extensively adopted in many branches of engineering to find optimal layouts that maximize target performances, see Refs. [1–3]. The conventional approach searches for the distribution of a prescribed amount of isotropic material such that the so–called compliance (twice the elastic strain energy computed at equilibrium) is minimized. A suitable interpolation can be adopted to penalize the mechanical properties of the elastic body depending on the local values of the unknown density field. In most cases, 0–1 solutions can be straightforwardly found implementing the well–known SIMP (Solid Isotropic Material with Penalization) [4]. Different methods are available in the literature to solve the so–called volume-constrained minimum compliance problem: among the others, one can use Optimality Criteria, see, e.g., [5] or methods of sequential convex programming such as CONLIN [6] and MMA [7]. All the above iterative approaches generally resort to the adoption of the finite element method to solve the equilibrium equation and compute the objective function and its sensitivity with respect to the design variables.

In general, most of the approaches for topology optimization deals with loads that are time–independent, with the main goal of optimizing a structure for an assigned set of constraints/supports. Pioneering contributions on the automatic placement of supports through structural optimization date back to the seventies, see e.g. Refs. [8–10], addressing beam supports, and [11], dealing with columns. Later, topology optimization of three–dimensional trusses including the cost of supports was investigated by Ref. [12] for both stiff structures and compliant mechanisms, whereas [13,14] tackled the optimal design of boundary conditions adopting spring supports at the nodes of the finite elements. The work in Ref. [15] introduced a design formulation attacking simultaneously the structural topology and the constraint locations, introducing new variables and enforcing a prescribed amount of allowable support. Afterwards, the work by Ref. [16] introduced a two–step procedure for the integrated layout design of supports and structures. Supports are intended as components that are partially embedded into the design domain and subjected to the applied boundary conditions. First, the optimal position of movable support components is found along a prescribed boundary of the design domain, then the layout optimization of the support components and the structure is performed. The above approaches cope with time–independent loads.

Several formulations exist to cope with the dynamic compliance of structures, see e.g. Refs. [17–20] but, to the authors' knowledge, no numerical method has been investigated yet to cope with the optimal design of supports in problems involving time dependent construction stages. Reference is also made to [21] that adopted parametric optimization to approach the mathematical modelling of time-dependent processes. For the sake of exposition, let us consider a specimen with a prescribed shape that is manufactured through a sequence of construction steps requiring the adoption of a suitable set of supports. The self–weight of the specimen is the prevalent design load that, in turn, depends itself on the evolution in time of the construction process.

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Hence, the bearing elements should be optimized to provide the stiffest support throughout the stages. This means that the compliance–based objective function should account not only for the final configuration, but also for *all* the intermediate shapes that are handled during the construction.

The need for the solution of the outlined design problem arises in many fields of applications, see, e.g., the construction of a bridge or the distribution of supports to perform 3D printing of complex shapes. Additive manufacturing, also known as 3D printing, nowadays is extensively used to create prototypes from digital models. Successive layers of material are laid down by a three–dimensional printer requiring support structures to sustain overhanging surfaces. Up to now, not any shape or geometry can be printed in real time, because a suitable set of supports must be engineered before synthesizing the three–dimensional object. Support structures remarkably affect not only the processing times but also the material consumption so their rationale design is crucial to improve the overall process of 3D printing. It must be finally remarked that additive manufacturing itself is a fertile area of research for topology optimization. In fact, 3D printing fills the gap between topology optimization and application, since any computed optimal design can be printed with minimal limitations on its complexity, see Ref. [22].

Goal of this work is to propose a new approach for the optimal placement (and design) of the supports in problems involving construction stages, thus including the inherent time–dependent nature of the process.

To this aim, a continuous optimization problem adopting a state equation defined as the time–integral of a linear elasticity problem on a space–time domain is formulated, while the objective function is given by the time-averaged compliance. The optimality conditions for this optimization problem are derived and a fixed-point algorithm is introduced for the iterative computation of the optimal solution. The equivalence between the *integral-in-time* formulation, used for the theoretical derivation of the optimality conditions, and a *pointwise-in-time* formulation of the state equation, exploited in the numerical approximation, is shown. The discretization of the optimization problem is finally obtained by considering n intermediate time instants t_i (and the corresponding spatial domains $\Omega(t_i)$) and solving a sequence of linear elasticity problems on $\Omega(t_i)$ with the finite element method. Numerical simulations obtained with this evolutionary topology optimization procedure have been firstly announced in Ref. [23], while in the present work we supply the theoretical framework in which the evolutionary continuous problem is defined as well as a detailed derivation of the resulting numerical scheme.

This work focuses on a topology optimization procedure that assumes linear elastic material and small displacements. Reference is made e.g. to [24–26] for robust methods to address problems involving geometric and material nonlinearities.

The outline of the paper is as follows. Sections 2 and 3 define the continuous and discrete topology optimization problems with the aim of designing the supports of an object exhibiting the minimum mean compliance over a time interval. Section 4 provides numerical examples showing the differences between a conventional SIMP method, which only optimizes the shape at the final time, and the proposed time–dependent approach. Section 5 provides final remarks on the presented methodology.

2. The continuous evolutionary topology optimization problem

In this section we describe the topology optimization problem which will be instrumental to optimally place the supports of the target object to be printed.

Horizontal layers of material are printed by additive manufacturing in subsequent time steps, to build the target object along with a suitable set of supports that cope with the evolving gravity load. The SIMP model is used to penalize the densities in the design domain, i.e. the region below the target object where the distribution of the support structures is unknown (Section 2.2). The inherent time–dependent nature of the process can be handled through the adoption of the time–averaged compliance as objective function (Section 2.3). Indeed, maximum stiffness is required not only at the end of the printing process, but throughout the process itself. A state equation defined as the time–integral of a linear elasticity problem on a space–time domain is adopted to formulate the optimization problem (Section 2.3). As in conventional minimum compliance problems, a limit on the available amount of material to build the supports is prescribed. The arising *integral-in-time* formulation can be handled to derive optimality conditions straightforwardly (Section 2.4), along with an efficient update scheme for the minimization unknowns (Section 2.5). Equivalence between the *integral-in-time* formulation and the more natural *pointwise-in-time* formulation for the state equation is also shown in Section 2.3.

2.1. Preliminaries

Let us consider an hold-all cylindrical space domain $\Omega = E \times (0, h) \subset \mathbb{R}^{d-1} \times \mathbb{R}^+$, with $d = 2, 3$ and E a subset of $\mathbb{R}^{d-1}$. Each point in Ω reads as $\mathbf{x} = (\mathbf{x}^*, y)$, where $\mathbf{x}^*$ denotes the planar component while y is the vertical one. Once the printing process is complete, i.e. for $t = T$, the target object $\mathcal{O}$ will occupy a certain subset $\Omega_1 \subset \Omega$, while for $t < T$ it will occupy intermediate configurations $\Omega_1(t)$ such that $\Omega_1(t) \subset \Omega$. In view of the above discussion, the value h represents the height of the object at the final time T. For future use, we also introduce the subdomain $\Omega_0 \subset \Omega$ identifying the region where *a priori* the user does not want to introduce any support. Next, we introduce a time-dependent domain $\Omega(t)$ that changes during the additive manufacturing process and represents the region where the 3D printer can add material (either belonging to the object or to the supports). We assume that $\Omega(t)$ grows in the direction given by the coordinate y with constant velocity V_0, i.e. $\Omega(t) = \{(\mathbf{x}^*, y) \in E \times (0, h) : 0 < y < V_0 t\}$. Accordingly, we have $\Omega_1(t) = \Omega(t) \cap \Omega_1$ (see Fig. 1). Clearly, at the final time $T = h/v_0$, we have $\Omega(T) = \Omega$ and $\Omega_1(T) = \Omega_1$.

In order to set up the topology optimization problem, we need to introduce a proper space-time domain and a suitable functional space. First, we define a proper space-time domain Q_T which is only a subset of $\Omega \times [0, T]$. This is motivated by the fact that at each time t we do not want to consider the whole Ω, but just a subset $\Omega(t)$. In view of this, we set (see Fig. 2)

$$Q_T = \cup_{t \in [0,T]} \{(\mathbf{x}, t) : \mathbf{x} \in \Omega(t)\}. \tag{2.1}$$

Then we introduce the functional space $\widetilde{\mathcal{V}}$ whose members are collections of displacement fields, one for each time in $[0, T]$. Let $\Gamma_D \subset \partial\Omega$ be the portion of the boundary where the object is anchored. In the following we assume for simplicity that Γ_D is a subset of the lower

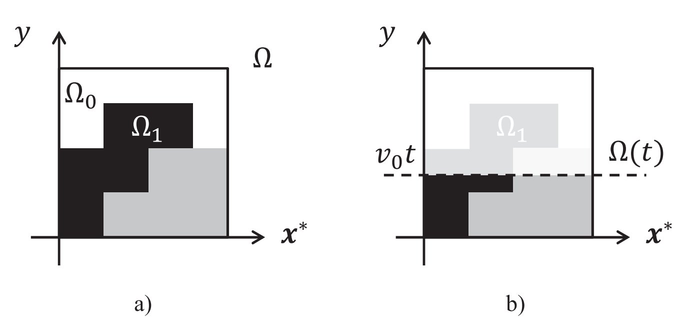

Fig. 1. a) Reference space domain. The domain occupied by the target object, i.e. Ω_1, is represented in black, the domain Ω_0 is represented in white, while the actual design domain $\Omega \backslash (\Omega_0 \cup \Omega_1)$ is represented in light grey. b) Space domain relative to time t. $\Omega(t)$ is located below the dotted line, corresponding to the height $v_0 t$).