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A consistent UVLWT formulation for laminated plane frame analysis considering semi-rigid connections

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A B S T R A C T

In this work we present a generalization of a finite element formulation, based on the new Unconstrained Vector Layerwise Theory (UVLWT), that allows a geometrically exact description of 2D laminated plane frames with accurate stress distribution. The formulation presents a natural and easy extension for 3D applications. The proposed generalization is based on a penalty strategy to couple frames at corners in a very accurate and elegant way. It consists on the introduction of rigid, semi-rigid and hinged connections among elements, for which all degrees of freedom of each lamina are considered and their energetically conjugate forces calculated. Flexible supports of the structure regarding the static reference have also been successfully developed and implemented.

Although two-dimensional frames are the main object of this study, three-dimensional connection expressions are developed in order to promote future developments and insights. In order to have complete solutions, even when a snap-back or snap-trough occur, we present the arc-length approach adapted to the complete analysis of 2D frames by the UVLWT.

From the analyzed examples, we can state that the UVLWT formulation and the developed connections presents the following capabilities:(i) geometric nonlinear modeling of laminated 2D frames assembled by slender or thick bars, without presenting problems of locking and system of equations bad conditioning; (ii) geometrically exact modeling of curved bars; (iii) consistent modeling of general flexible or rigid internal corner connections; (iv) consistent modeling of elastic external supports; (v) Zig-Zag effect consideration with accurate transverse stress distribution and (vi) straightforward extension for 3D applications as the finite rotation concept is not used.

1. Introduction

Laminate is a composite material formed by the bound overlapping of two or more laminas (flat or curved) of different materials, composite or not.

The arrangement of laminas occurs according to a stacking scheme whose possibilities are: the number and thickness of laminas, the stacking sequence and the orientation of the orthotropic behavior of each lamina [1]. Laminates are structural composites designed mainly for applications requiring low weight and high mechanical performance [1]. Consequently, its use is largely spreading in the most different engineering areas.

In the analyses of structures constituted by laminates, applying the macromechanic approach is considered a convenient and simple approach to achieve the global effects, as displacements, internal efforts, natural frequencies and buckling critical loads. The strategy most adopted to solve engineering problems following the macromechanic approach is to link the Finite Element Method (FEM) with laminate theories. These theories allow determining the behavior of the laminate as function of the individual properties of each lamina and the stacking scheme [2]. Among the laminate theories one may detach the *Equivalent Single Layer* (ESL) and the *Layerwise* theories [3].

Formulations based on the ESL theory are the simplest, they consider the laminate as a single equivalent homogeneous lamina [4]. However, the real behavior of laminates presents discontinuous transverse strains at laminas interfaces due to transverse heterogeneity and the interlaminar continuity of the transverse stresses. From this reasoning, the displacement field presents discontinuity of derivatives characterizing the so called Zig-Zag effect [5].

The ESL theories, considering the laminate as a single equivalent lamina, do not represent the Zig-Zag effect and, therefore disrespect the interlaminar continuity of the transverse stresses. The equivalent theories have limited application to accurately analyze thin, thick and laminates with large variation of the elastic properties among laminas, because they

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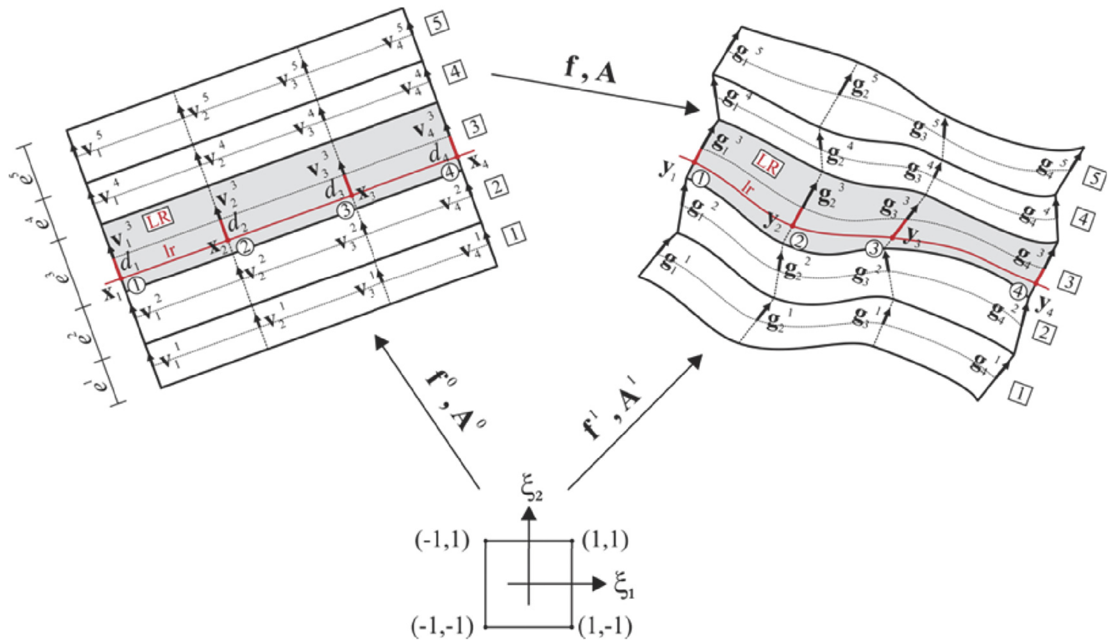

Fig. 1. General position mapping - five laminas illustration.

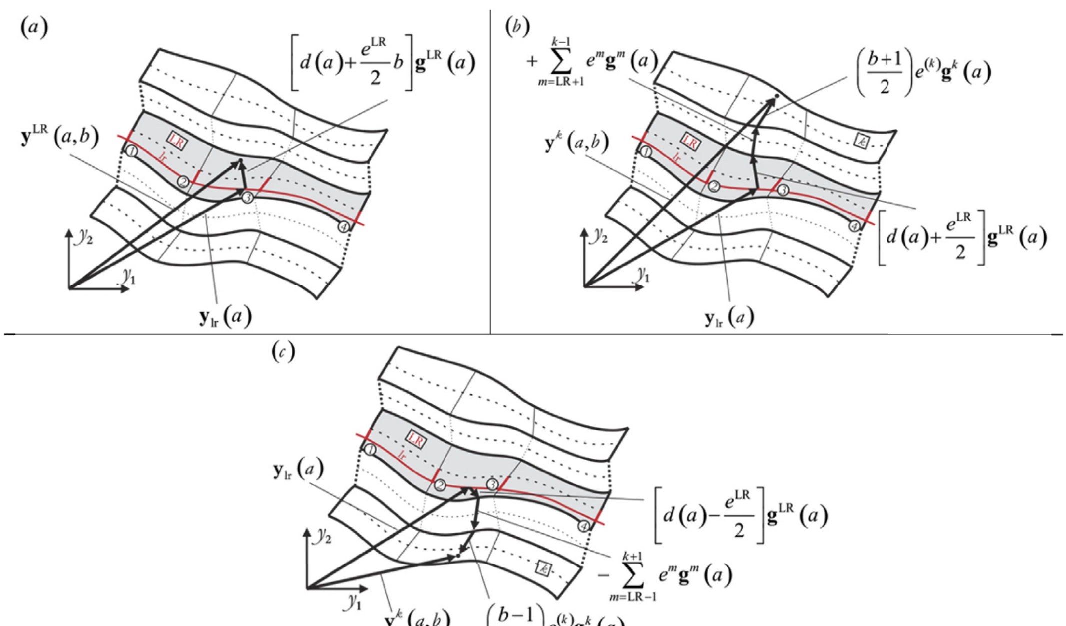

Fig. 2. Mapping of a pont with dimensionless variables ($\xi_1 = a, \xi_2 = b$)located inside laminas: **(a)** LR, **(b)** above LR and **(c)** under LR.

do not provide a realistic stress state at the lamina level and at its interfaces [5]. This statement is also valid when there is an interest in representing material failure, for example, delamination and slipping, which are the most important types of failure in laminated composites.

In these circumstances, formulations based on Layerwise theories are the most employed [2,3,6], because they can represent the Zig-Zag effect and make it possible to satisfy the interlaminar continuity of stresses [5,7,8]. The basic idea of Layerwise theories is to consider each lamina independently, but to impose displacements and/or stresses constraints on lamina's interface.

Finite elements developed using Layerwise theories present the power of three-dimensional models, hold the characteristics of the ESL formulations regarding mesh generation and reduced computational cost [9].

However, problems of bad conditioning of the system of equation and locking phenomena can arise when problems whose thickness of the laminate is much smaller than the other dimensions, or when there is a large variation in the elastic properties of laminas [5].

As mentioned before, the laminated composites have high specific strength and stiffness, making possible the design of light and slender structures [3]. Therefore, the idea that laminates are used in secondary structures, leaving the main structures for traditional materials, is quickly changing [10]. Consequently, nowadays the consideration of geometric non-linear effects in structural analyzes is mandatory.

Regarding the application of the FEM in geometric nonlinear analysis, spatial description (Eulerian) or material description (Lagrangian, total