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A finite element/quaternion/asymptotic numerical method for the 3D simulation of flexible cables

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ABSTRACT

In this paper, a method for the quasi-static simulation of flexible cables assembly in the context of automotive industry is presented. The cables geometry and behavior encourage to employ a geometrically exact beam model. The 3D kinematics is then based on the position of the centerline and on the orientation of the cross-sections, which is here represented by rotational quaternions. Their algebraic nature leads to a polynomial form of equilibrium equations. The continuous equations obtained are then discretized by the finite element method and easily recast under quadratic form by introducing additional slave variables. The asymptotic numerical method, a powerful solver for systems of quadratic equations, is then employed for the continuation of the branches of solution. The originality of this paper stands in the combination of all these methods which leads to a fast and accurate tool for the assembly process of cables. This is proved by running several classical validation tests and an industry-like example.

1. Introduction

During the last decades, the room available in car vehicles (e.g. in engine compartment) has plummeted because of the rapid development of on-board electronics. As a result, a need for very accurate numerical tools for design has appeared in automotive industry. In the meantime, a fast computation is necessary so that design duration remains suitable for industry. In this context, flexible pieces represent an outstanding challenge since, unlike most of car pieces, they cannot be modeled as rigid body solids in CAD software. This paper focuses on a specific type of flexible piece, namely electrical cables. Cables have a complex structure. A wire is made up of copper filaments wrapped in an elastomer duct. These wires are most of the time gathered in bundles which are themselves surrounded by various protections such as tape, PVC tube … Moreover, the full cable is often constituted of several drifted cable pieces forming a system with a complex geometry.

Due to its slender shape and its flexibility, one can consider a simple cable as a beam undergoing large displacements and large rotations. There exist several theories accounting for nonlinear beams, see Refs. [1–3]. The most widely used theory is the geometrically exact beam model whose founding principles were established by Reissner [4,5] and further generalized by Simo [6]. Various finite element formulations (FEM) have been presented to solve these equations numerically. The notable works of [7–11] and more recently [12,13] can, among others, be listed.

At the heart of all these formulations, the rotation parameterization is of paramount importance in the numerical models. The 3D rotations modeling indeed is not an easy task especially when computational efficiency is sought. The rotational vector-like parameterization used by many authors features only 3 parameters (minimum set in 3D). However, as no parameterization of less than 4 parameters can be singularity-free [14], this choice poses several numerical limitations and lacks robustness. A powerful alternative consists in using quaternions, a set of 4 singularity-free parameters. Firstly used only for storage in the numerical models, Zupan et al. [12] have recently shown their utility when used as primary variables. They also have developed a model without rotation matrices exploiting the high potential of quaternion algebra, and very efficient for numerical purposes [15]. In addition, quaternions offer an original description of rotations since they substitute the usual trigonometric functions by algebraic variables and lead to polynomial equilibrium equations.

The finite element method, very adapted to the assembly of complex geometries such as electrical cables ones, is applied to the continuous equilibrium equations. The algebraic system obtained is then

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generally solved by a classical predictor-corrector method (PCM) [16]. Even if the appearance of arc-length methods [17] has considerably enhanced the robustness of this type of method, they often require to choose a step size which may be very tricky for the user. A sufficiently small step size allows to compute even highly nonlinear part of equilibrium branches but in return may impractically increase the computation time, while a larger step size may spoil the convergence. Taking advantage of the polynomial form of the system of equations obtained when using quaternion parameters an alternative consists in replacing the PCM by the asymptotic numerical method (ANM) firstly presented by Damil and Potier-Ferry [18] and by Cochelin [19]. The ANM is a very powerful solver for quadratic problems and it overcomes all the drawbacks of the PCM. This technique indeed is very robust, does not require any tuning parameters and is thus well suited for an industrial use. In addition, Cochelin and Medale [20] have equipped the method with a bifurcation detector and improved its efficiency in the vicinity of bifurcation points.

Combining quaternions with the ANM has already been set up on a rod model discretized with a finite difference scheme by Lazarus et al. [21], which have got very promising results. We propose here to set up the technique on the finite-element based geometrically exact beam model, in what constitutes the main originality of this paper. Validations and illustrations of the method on very intricate problems are provided and discussed. A critical evaluation and future researches are presented by way of conclusion.

2. Governing equations

In this section, the classical quasi-static formulation of the geometrically exact beam model, based on the rotation vector, is firstly presented. It enables to explain all the main ingredients of the model and to discuss their physical meaning. Secondly, the equations are modified by using quaternions instead of the rotation vector. This leads to the formulation which serves our numerical model, presented in part 3.

2.1. The geometrically exact beam model

In the geometrically exact beam model, a beam is described by defining a family of cross-sections whose centroids form a curve called the centerline of the beam. The kinematic variables essential to this description are introduced following the notations used in Ref. [22]. Let us consider a beam of initial length L, with an arbitrary cross-section Ω, as depicted Fig. 1. Let $(\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3)$ be a fixed Cartesian (global) frame. The current position of the centerline in this frame is described by the vector field $\boldsymbol{x}_0(s)$ which is a function of the curvilinear abscissa $s \in [0, L]$ along the beam axis. A material (local) frame $(\boldsymbol{e}_1(s), \boldsymbol{e}_2(s), \boldsymbol{e}_3(s))$ is introduced to define the cross-section at abscissa s. The vectors $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ span the cross-section while the vector $\boldsymbol{e}_1$ remains normal to the cross-section for every deformed configurations. It is essential to note that $\boldsymbol{e}_1$ is not necessarily tangent to the centerline of the beam such that shear deformation is taken into account (Timoshenko model). The reference configuration is chosen such that the beam is unstressed. However, in this state, the reference position of the centerline is an arbitrary curve and not necessarily a straight line, thus accounting for the initial curvature of the beam. Its initial position is then a function of s denoted $\boldsymbol{X}_0(s)$, such that the displacement of any point of the centerline is $\boldsymbol{x}_0(s) - \boldsymbol{X}_0(s)$. Similarly, the material frame in the reference configuration depends on s and is denoted $(\boldsymbol{E}_1(s), \boldsymbol{E}_2(s), \boldsymbol{E}_3(s))$. To end up with the definition of the kinematic variables, let us introduce the two rotation operators $\boldsymbol{R}_0(s)$ and $\boldsymbol{R}(s)$ which depict the orientation of the material frames in the global frame in the initial and in the current configuration respectively, that being $\boldsymbol{E}_i(s) = \boldsymbol{R}_0(s)\boldsymbol{u}_i$ and $\boldsymbol{e}_i(s) = \boldsymbol{R}(s)\boldsymbol{u}_i$, $i \in \{1, 2, 3\}$. As $\boldsymbol{R}_0(s)$ defines the initial curvature of the beam it is constant through the deformation. With these notations, the position vector in the spatial frame of any point M' of the undeformed beam located in the section at s writes

$$\boldsymbol{X}(s, X_2, X_3) = \boldsymbol{X}_0(s) + \boldsymbol{R}_0(s)\boldsymbol{Y}(X_2, X_3), \tag{1}$$

where $\boldsymbol{Y}(X_2, X_3) = [0 \quad X_2 \quad X_3]^T$ is the material position of M' in the cross-section. Under the assumption that cross-sections remain plane and do not undergo any deformations along the transformation, $\boldsymbol{Y}(X_2, X_3)$ is constant and M' after deformation becomes M'' whose position is given by

$$\boldsymbol{x}(s, X_2, X_3) = \boldsymbol{x}_0(s) + \boldsymbol{R}(s)\boldsymbol{Y}(X_2, X_3). \tag{2}$$

The current configuration of the beam is then completely characterized by the position of the centerline $\boldsymbol{x}_0(s)$ and the orientation of the cross-sections $\boldsymbol{R}(s)$. One recovers here that the kinematic variables depend solely on the curvilinear abscissa s as for any beam model. As it is

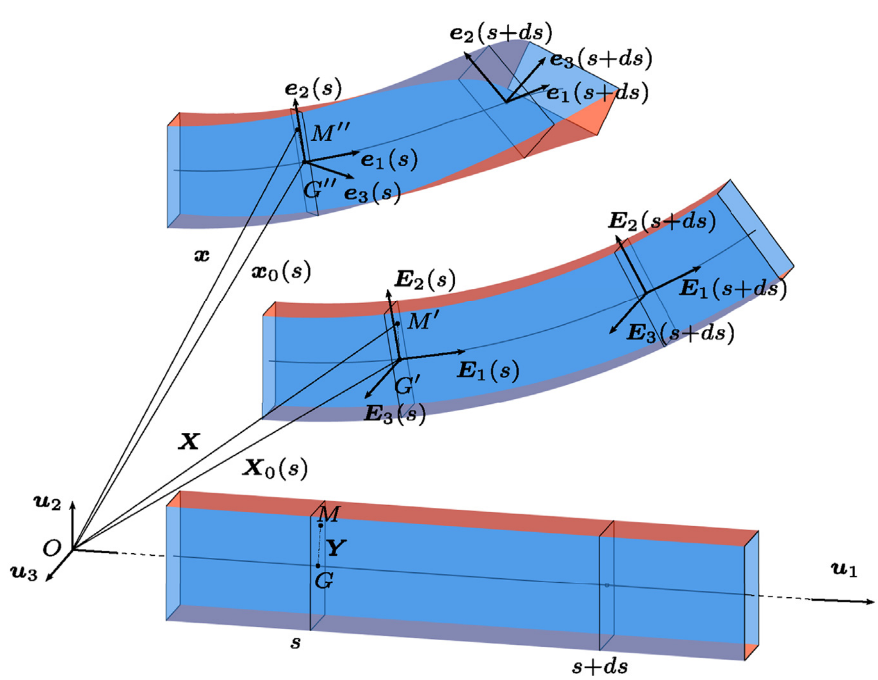

Fig. 1. Kinematics of the geometrically exact beam model.