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Incremental volumetric and Dual Kriging remapping methods

C. Miguel [a,*], D.M. Neto [a], M.C. Oliveira [a], J.L. Alves [b], L.F. Menezes [a]

[a] CEMMPRE, Department of Mechanical Engineering, University of Coimbra, Polo II, Rua Luís Reis Santos, Pinhal de Marrocos, 3030-788, Coimbra, Portugal
[b] CMEMS, Microelectromechanical Systems Research Unit, University of Minho, Campus de Azurém, 4800-058, Guimarães, Portugal

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ABSTRACT

The transfer of variables between distinct spatial domains is a problem shared by many research fields. Among other applications, it may be required for visualization purposes or for intermediate analysis of a process. In any case, two important factors must be considered: accuracy and computational performance. The accuracy becomes more important when the results have an impact on the subsequent stages of the process' analysis, as it could lead to incorrect results. The computational performance is a permanent requirement due to the ever-increasing complexity of the analysed processes. The aim of this work is to present a new remapping method, based on Dual Kriging interpolation, developed to enable accurate and efficient variable transfer operations between two different domains, discretized with hexahedral finite elements. Two strategies are proposed, which take into account different selections of interpolation points and are based on specific Finite Element Method features. They are compared with the Incremental Volumetric Remapping method in two remapping examples, one of which includes a trimming operation, highlighting their advantages and limitations. The results show that the Dual Kriging remapping method, combined with a 2D selection strategy for the donor points, can contribute to increase the accuracy of the state variables remapping operation, particularly when they present a strong gradient along the stacking direction.

1. Introduction

The Finite Element Method (FEM) emerged in the sixties, enabling the solution of problems that could not be solved analytically [1,2]. Whatever the application field, FEM requires the partition of the spatial domain into finite elements, which define the mesh that approximates the original domain. The type of finite elements used depends on the application field or problem (e.g. Ref. [3]), and varies between 1D, 2D, and 3D, using interpolation functions of different degrees. The linear isoparametric hexahedral and tetrahedral finite elements are commonly used in 3D simulations, adopting different mesh topologies.

The solution of nonlinear problems also requires the division of the temporal domain to take into account time dependant variables related to geometrical, material or boundary conditions nonlinearities [4]. Typically, resorting to more divisions – spatial and temporal – increases the accuracy of the numerical results, at the cost of the computational performance (e.g. Refs. [5,6]). Therefore, it is always necessary to find the best equilibrium between results accuracy and computational effort. In terms of spatial discretization, the definition of zones with different mesh sizes can be performed either in the pre-processing stage or during the numerical simulation. The definition of different zones in the pre-processing is usually carried out manually, which contributes to an increase of the time required for this stage. On the other hand, during the numerical simulation, the definition of different zones requires the application of adaptive mesh refinement/remeshing algorithms (e.g. Refs. [7,8]), in several temporal increments. In fact, adaptive mesh refinement algorithms are commonly used to overcome problems of excessive distortion/deformation of the finite elements, which occur in different forming processes [7,9], such as forging [10,11] and sheet metal forming [12]. The adaptive mesh refinement is usually performed by one of three methods: *p*-adaptive (change of the interpolation degree), *h*-adaptive (change of the element size), *r*-adaptive (change of the nodes' location); or by a combination of them [8]. The improvement of these methods is an up to date research topic in computational mechanics [13], since small improvements have a considerable impact on computational performance, as they can be applied several times during the simulation.

The adoption of adaptive mesh refinement algorithms involves a remapping step, i.e. the transfer of variables between different spatial discretizations [14,15], which can present a strong influence on the accuracy of the results and computational efficiency. Nonetheless, in some

* Corresponding author.
E-mail addresses: carlosmiguelpapers@gmail.com (C. Miguel), diogo.neto@dem.uc.pt (D.M. Neto), marta.oliveira@dem.uc.pt (M.C. Oliveira), jlalves@dem.uminho.pt (J.L. Alves), luis.menezes@dem.uc.pt (L.F. Menezes).

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cases the zone to be remapped can be narrow, as in the numerical analysis of trimming operations involved in some multi-stage forming processes. Typically, these operations consist in the geometrical trimming of the FE mesh (e.g. Ref. [10]), which is also the focus of this work. Thus, after each trimming operation, it is also necessary to perform a remapping procedure for the numerical variables involved in the subsequent forming steps. In this case, the impact of the selected remapping method on the computational efficiency is reduced, but its accuracy can have a strong influence on the results of the subsequent numerical simulation. When using the FEM, it can be necessary to transfer the nodal variables (primary unknowns), such as forces and displacements; and the state variables (secondary unknowns), which are evaluated in the integration points (typically Gauss points), such as the stress and strain state. The former are usually continuous while the latter are discontinuous (e.g. Ref. [7]). The current work is mainly focused on the transfer of state variables, evaluated in the integration points.

Jiao and Health [14], divide the remapping methods into four major groups. The first group refers to pointwise interpolation and extrapolation methods, such that the variables are transferred using a function that interpolates/extrapolates the variables from the donor (old) mesh to the target (new), in one or more stages. The pointwise interpolation can be categorized into two types: (i) use of the same shape function as the one for the donor mesh (sometimes referred to as consistent interpolation or inverse isoparametric mapping (e.g. Refs. [16,17])) and (ii) use of basis functions of higher order than the one of the donor basis. According to Baptista [18], taking into account the mathematical characteristics of these methods, this group can also include the ones based on the application of the moving least squares [15,19] and the Superconvergent Patch Recovery methods, developed by Zienkiewicz and Zhu [20]. The second group refers to Area/Volume weighted averaging methods (also referred as Finite Volume Transfer Method in Ref. [11]), which uses a transfer function that is evaluated based on the area/volume of intersection between the donor and the target FE. The corresponding areas/volumes act as a weighting factor defining the contribution of each donor element to the target one (e.g. Refs. [21,22]). The third group refers to Mortar element methods, which are general techniques for projecting data at interfaces between two or more non-conforming subdomains [11]. From a mathematical point of view, this method consists in the minimization of a weighted residual, where the weight functions are usually chosen from the space spanned by the basis functions of the mortar side [23]. The last group refers to specialized methods, which are designed for specific applications and do not fall directly into the above categories, but frequently are variants or combinations of them. This fourth group includes the direct allocation to the target point of the closest donor point [24], the use of different methods according to the type of variable [9], and adaptations of the interpolation/extrapolation and area/volume weighted averaging methods; by including constraints [25]; and/or considering specific features of the application domain or problem [26,27].

The accurate transfer of variables between different spatial discretizations is imperative, independently of the remapping method adopted. Moreover, its computational performance is particularly important when the procedure is performed several times, while the error is accumulated to the subsequent stages (e.g. Refs. [7,11]). In fact, the remapping operation can introduce errors due to the approximations used to estimate the values for the target mesh. In order to try to control and minimize the unavoidable errors when performing remapping operations, several authors [11,14,15,17,25] point out some desirable characteristics. The method should be self-consistent such that when the target and the donor point are coincident, the transfer remapping function reduces to the identity operator (null error). The interpolation/extrapolation methods that resort to the donor shape functions cannot guarantee, *a priori*, this condition [15]. On the contrary, Area/Volume weighted averaging methods automatically verify this self-consistency condition. The method should also guarantee the locality, i.e. the remapped value in a target point should only be affected by the variables of the donor mesh in a region of influence. This assures the preservation of discontinuities, related with material or geometric interfaces, which must also be present in the remapped mesh. However, due to the discrete and approximated natured of the remapping operations, it is always expected some degradation (smoothing) of the variable value when severe gradients are present. Nonetheless, the smoothing should be minimized in order to preserve, as accurately as possible, the gradients of the donor mesh. On the other hand, the remapping method can also lead to spurious local extreme values, which are non-physical and result in the degradation of the numerical simulation result. Accordingly, the remapping algorithm should allow for the inclusion of some constraints, such as consistency of equilibrium or motion equations [25], consistency between the displacement field and the stress state or boundary conditions [17].

In order to take advantage of the characteristics of these applications, several specialized remapping methods have been proposed, in order to find the best equilibrium between accuracy and computational performance. One example is the application of the moving least squares method, proposed by Rashid [15], based on a transfer function. It attempts to force the equality between the variable field in a volumetric domain of the donor and of the target mesh, assuming that each donor integration point has constant state variables in a predefined region. Jiao and Heath [14] present a general method, named Common Refinement, which is based on the intersection of the donor and target mesh in order to define a third mesh, used as an auxiliary for the transfer procedure. The main advantage of this method is that it allows the accurate integration of the transfer function, which depends on the shape functions of the target and donor meshes. However, it requires a robust and expeditious algorithm for mesh intersection, which is considerably challenging to attain when working with solid hexahedral finite elements. In this context, also the Incremental Volumetric Remapping (IVR) method was developed and applied, specifically for the transfer of variables between meshes composed of linear isoparametric hexahedrons [18]. This volume-weighted averaging method assumes that each donor integration point has constant state variables in a predefined region [15]. Being a volume-weighted averaging method, some of the desirable characteristics are inherently verified (self-consistency, locality, and inexistence of spurious local extrema values), which makes it particularly interesting for FEM analysis.

The IVR method has been previously implemented in the in-house code DD3TRIM [18], which has been specifically developed for performing geometrical trimming operations of 3D meshes, composed by linear isoparametric hexahedrons. In sheet metal forming operations, this type of elements is typically used with a selective reduced integration scheme [28]. Thus, for each element, the state variables are evaluated in eight different integration points, also called Gauss Points (GPs), since their spatial positions in the finite element's natural coordinates are defined by the Gauss Quadrature Rule [4]. The accurate transfer of the state variables is fundamental to enable the proper numerical simulation of forming process, involving trimming operations.

In previous works [18,29,30], the performance of the IVR algorithm was compared with the classic interpolation/extrapolation method, using a transfer function based on the shape function of the linear isoparametric elements. Additionally, it was compared with the moving least square interpolation method, using an exponential based curve as weight function [18,29]. The results show that the error associated to the IVR is lower when compared to these other two methods, particularly when increasing the number of consecutive remapping operations. In addition, the IVR method is robust in critical situations, such as poor geometrical definition of the mesh domain boundaries, where some nodes of the target mesh fall outside the donor mesh. However, concerning the computational cost, it was observed that the classical extrapolation/interpolation method was clearly the fastest, while the IVR method and the one based on moving least squares interpolation presented similar computational costs [18,29].

This study presents a new remapping method dedicated to finite