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Simulation-based prediction of cyclic failure in rubbery materials using nonlinear space-time finite element method coupled with continuum damage mechanics

Shogo Wada [a], Rui Zhang [a], Seetha R. Mannava [b], Vijay K. Vasudevan [b], Dong Qian [a,*]

[a] Department of Mechanical Engineering, University of Texas at Dallas, Richardson, TX 75080, USA
[b] Department of Mechanical and Materials Engineering, University of Cincinnati, Cincinnati, OH 45221-0072, USA

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ABSTRACT

Rubbery materials are widely used in industrial applications and are often exposed to cyclic stress and strain conditions while in service. To ensure safety and reliability, quantifying the effect of loads on the life of rubbery material is an important but challenging task, due to the combination of geometric/material nonlinearities and loading conditions for extended time durations. In this work, a novel simulation approach based on nonlinear space-time finite element method (FEM) is presented with a goal to capture fatigue failure in rubbery material subjected to cyclic loads. It is established by integrating the time discontinuous Galerkin (TDG) formulation with nonlinear material constitutive laws. A continuum damage mechanics (CDM) model is introduced to account for the damage evolution and model parameters for synthetic rubber are calibrated based on experiment. The nonlinear space-time FEM coupled with CDM constitutive model shows good agreement with the fracture and low cycle fatigue test of notched rubber sheet specimen.

1. Introduction

Rubbery materials are widely used in industrial products such as tire, engine mount, seismic isolation materials, sealing, medical equipment etc. The special properties of rubber (large deformation, incompressibility, rate-temperature dependency, stress-strain hysteresis, etc.) cannot be easily replaced by other materials. Under operating environments, parts made of rubber can be subjected to cyclic stress and strain conditions over extended time period. As such, durability is an important aspect in the design of rubber. As an example, a standard automotive tire is a composite that consists of rubber, steel wire and textile and the resultant stress-strain history inside the tire is complex. Failure modes of the tire that initiate from the edge or interface of the reinforcements have been reported [1]. A general review on fatigue failure in rubbery materials can be found in Mars and Fatemi [2].

Fatigue life prediction in rubbery materials is a challenging task due to the lack of understanding on the controlling mechanisms. Most of the life prediction tools employed by the industry today are heavily empirical in nature. They can be broadly classified into two categories based on the parameters chosen to calculate fatigue life, i.e., models based on fatigue crack initiation/propagation (FCIP) and cumulative fatigue damage (CFD). A comprehensive review on the fatigue life prediction models for rubbery materials can be found in Mars and Fatemi [3]. In FCIP approach, maximum principal strain or strain energy density are widely used as the criteria for fatigue failure. As an example, Fielding [4] showed the relationship between uniaxial strain and fatigue life of synthetic rubber. On the other hand, fatigue crack growth approach is mainly based on strain energy release rate [5,6] or J-integral [7]. Mars and Fatemi [3] summarized both crack nucleation and growth approaches. In CFD approach, continuum damage mechanics (CDM) theory is introduced in which it is assumed that the internal damage variable is accumulated due to cyclic load. Lemaitre [8] developed this concept for ductile failure of metals. Cantournet and Desmorat et al. [9,10] applied CDM for modelling Mullins effect [11] and cyclic softening of elastomers. For the fatigue life of rubber, Wang et al. [12] proposed CDM model as a function of strain amplitude.

To handle large and highly nonlinear deformation of rubbery materials while subjected to cyclic loads, numerous constitutive models have been established [13]. Isotropic hyperelastic model have been commonly introduced for rubber in which strain energy is expressed as a function of the three principal invariants of the Cauchy Green deformation tensor, given as I_1, I_2 and I_3. In this context, Neo-Hookean material model is the

* Corresponding author.
E-mail address: dong.qian@utdallas.edu (D. Qian).

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simplest form and can be regarded as an extension of Hooke's law to large deformation. A polynomial form involving the principal invariants was introduced in Rivlin and Saunders [14]. Mooney-Rivlin model [14,15] is also a standard model that has been widely used for the modeling of rubber. It is described as a combination of first order terms of I_1 and I_2. In Yeoh's model [16], strain energy potential is a cubic form of the first invariant. Ogden [17,18] extended the polynomial form of the strain energy by using summation of non-integer order principal stretches. Arruda and Boyce [19] proposed the stretch based model known as the 8-chain model. Because of incompressibility of the rubber, the volume ratio is theoretically assumed to be one in these hyperelastic models. To enforce the incompressibility constraint, methods of modifying the hyperelastic models have been developed and used [20–22]. In some applications, viscoelastic properties of rubber must also be considered. Simo [23] developed finite strain viscoelastic theory by extending the linear viscoelasticity model. With this formulation, arbitrary hyperelastic material models can be incorporated.

While a great deal of efforts has been devoted to fatigue modeling, relatively little progress has been made towards direct numerical simulation (DNS) of fatigue failure in rubbery materials. The challenges in establishing the DNS tools are mainly due to the temporal scales associated with the application. Fatigue failure in rubber may take from a few thousands to millions of load cycles, Traditional computational tool such as the finite element method (FEM) based on semi-discrete schemes is not well suited for these types of analysis as it lacks the flexibility in establishing approximations in the temporal domain. Semi-discrete time integration schemes such as the center difference or Newmark-β methods are known to suffer from either the time-step constraints or lack of convergence due to the oscillatory nature of the fatigue loading condition. As such, simulating loading conditions with cycles on the order of hundreds of thousands and beyond is generally an impractical task for FEM. On the other hand, there is a great demand for such a computational capability as factors such as stress history and triaxiality, nonlinear coupling among the loads, complex geometry are known to critically influence the fatigue failure in rubber and generally not fully accounted for in the empirical design approaches that are in practice today.

Motivated by the current development, a novel nonlinear space-time FEM based on the time discontinuous Galerkin (TDG) formulation [24–26] is established in this paper with a goal to capture failure in rubbery material while subjected to cyclic load. Unlike conventional FEM that employs the semi-discrete scheme, shape functions are introduced in both the spatial and temporal domain in space-time FEM, thus enabling extended capabilities in simulating responses that are strong, nonlinear functions of time. Use of TDG formulation effectively decomposes the spatial-temporal domain into multiple space-time slabs that are coupled through the jump terms, which significantly reduces the computational cost. Further it has been shown that TDG based formulation is both A-stable and higher-order accurate [24,25]. Extension of TDG formulation to solid mechanics problems were presented in Hulbert and Hughes [25], Hughes and Stewart [27], Li and Wiberg [28]. These works have demonstrated that TDG formulation significantly reduces the artificial oscillations that are commonly associated with semi-discrete time integration schemes in capturing sharp gradients or discontinuities. More recently, it has been proposed that the convergence properties of the regular space-time FEM can be further enhanced by introducing enrichment functions that represent the problem physics [29,30]. This enriched formulation is referred to as the extended space-time finite element method (XTFEM) [31]. The robustness of XTFEM has been demonstrated in the context of coupled atomistic-continuum simulations of fracture [32], wave dynamics [33] and high cycle fatigue failure in metals [34,35].

The main objective of this work is to establish a nonlinear TDG-based space-time approach to failure prediction of rubbery materials. Motivation for the space-time approach is to overcome the limitations associated with the semi-discrete scheme in the finite element method. We note that most of the prior works summarized earlier on TDG have focused on the linear formulations, and nonlinear TDG-based space-time FEM has not been systematically established and applied for practical application. In this work, the Mooney-Rivlin and CDM models are coupled with TDG formulation. This integration leads to a nonlinear space-time FEM implementation that incorporates both geometric and material nonlinearities.

The rest of the paper is organized as follows: Section 2 presents the basic formulation and implementation of nonlinear space-time FEM based on TDG. The constitutive models employed for rubber and their integration with the space-time FEM are reviewed in section 3. The robustness of the proposed approach is demonstrated through numerical examples in Section 4 along with verifications and validations against experiments and finite element method. Finally, conclusions and summary are provided in section 5.

2. Nonlinear space-time FEM formulation

2.1. Nonlinear space-time FEM formulation

Time discontinuous Galerkin (TDG) formulation belongs to the general family of discontinuous Galekin method that has been introduced by Reed and Hill [36], Lesaint and Raviart [37] and application to elastodynamics can be found in Refs. [25,27,28,38]. The basic formulation of space-time FEM based on TDG formulation with linear elasticity is described in Refs. [25,34]. In this section, an extension is made to the case of nonlinear problems. Here we adopt the total Lagrangian formulation [39]. The strong form of initial/boundary value problem is defined over spatial domain Ω_0 and temporal domain $I =]0, T[$ and is given as follows.

$$\nabla_{\mathbf{X}} \cdot \mathbf{P} + \rho_0 \mathbf{b} = \rho_0 \ddot{\mathbf{u}} \quad \text{on } Q \equiv \Omega_0 \times I. \tag{1}$$

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \gamma_u \equiv \Gamma_u^0 \times I. \tag{2}$$

$$\mathbf{n}_0 \cdot \mathbf{P} = \mathbf{t}_0 \quad \text{on } \gamma_t \equiv \Gamma_t^0 \times I. \tag{3}$$

$$\mathbf{u}(\mathbf{X}, 0) = \mathbf{u}_0(\mathbf{X}) \quad \text{on } \mathbf{X} \in \Omega_0. \tag{4}$$

$$\mathbf{v}(\mathbf{X}, 0) = \mathbf{v}_0(\mathbf{X}) \quad \text{on } \mathbf{X} \in \Omega_0. \tag{5}$$

Here $\mathbf{X}$, $\mathbf{u}$, $\mathbf{P}$, $\mathbf{b}$ and ρ_0 represent the material coordinate, displacement, nominal stress tensor, body force and mass density, respectively. Superimposed dot indicates the time derivative, $\bar{\mathbf{u}}$ and $\mathbf{t}_0$ are the prescribed displacement and traction over the essential boundary Γ_u^0 and nature boundary Γ_t^0, respectively. Further, we have $\Gamma^0 = \Gamma_u^0 \cup \Gamma_t^0$ and $\Gamma_u^0 \cap \Gamma_t^0 = \varnothing$. Finally, $\mathbf{u}_0$ and $\mathbf{v}_0$ are the initial displacement and velocity.

In TDG formulation, the temporal domain $I =]0, T[$ is divided into multiple sub-domains $I_n =]t_{n-1}, t_n[$ and each sub-domain and spatial domain are combined as a space-time slab $Q_n = \Omega_0 \times I_n$. In addition, essential and traction boundary conditions are defined on $(\gamma_u)_n = \Gamma_u^0 \times I_n$ and $(\gamma_t)_n = \Gamma_t^0 \times I_n$ respectively. Furthermore, a space-time slab Q_n is discretized into $(n_{el})_n$ space-time elements $Q_n^e \subset Q_n$ and its boundary is γ_n^e. Fig. 1 provides an illustration of the space-time discretization.

We introduce the jump operators to treat the temporal jumps between the neighboring space-time slabs:

$$[\![\mathbf{u}(t_n)]\!] = \mathbf{u}(t_n^+) - \mathbf{u}(t_n^-), \tag{6}$$

$$\mathbf{u}(t_n^{\pm}) = \lim_{\varepsilon \to 0^{\pm}} \mathbf{u}(t_n \pm \varepsilon). \tag{7}$$

In deriving the weak form in TDG formulation, we first introduce the trial functions $\mathbf{u}^h(\mathbf{X}, t)$ and test functions $\delta \mathbf{u}^h(\mathbf{X}, t)$ that are C^0 continuous within each space-time slab. Trial and test functions can have discontinuities across the space-time slabs. The spaces of the trial function and