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Nonlinear dynamic analysis of creased shells

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ABSTRACT

Recent studies analyze the behavior of advanced shell structures, like foldable, multistable or morphing shell structures. Simulating a thin foldable curved structure is not a trivial task: the structure may go through many snapping transitions from a stable configuration to another. Then, one could claim arc-length methods or use a dynamic approach to perform such simulations. This work presents a geometrically exact shell model for nonlinear dynamic analysis of shells. An updated Lagrangian framework is used for describing kinematics. Several numerical examples of folding a thin dome are presented, including creased shells. The triangular shell finite element used offers great flexibility for the generation of the unstructured curved meshes, as well as great results.

1. Introduction

Curved shells can be found in many natural and man-made structures. Shells have high structural efficiency due to their curvature, as bending and stretching are coupled to handle deformations, making them energetically costly to deform. Due to their high strength/weight ratio, ability to shelter inner components and a good esthetic value, shells are important structures for engineering applications. Many examples can be listed: pipes, beer cans, eggshells, skulls, bells, bowls, tents, corneas, lens, wine glasses, tanks, silos, domes, roofs, structures of airplanes, submarines, ships, rockets, missiles, etc.

The study of shells dates back to the nineteenth century, when Love [1] presented important contributions to the thin shell theory. He applied the Kirchhoff's assumptions [2], originally derived to thin plate bending theory, to the shell theory, together with the assumptions of small deflection and small thickness of the shell. Similar first order approximation shell theories were presented by Donnell [3], Sanders [4] and Flügge [5].

A second order approximation shell theory was derived by Reissner [6,7], where the assumptions on the preservation of the normals and that the transverse normal strain may be neglected were abandoned, thus considering the deformations caused by the transverse shear forces.

The geometrically nonlinear shell theory had considerable contributions by authors like Mushtari [8], Sanders [9], Naghdi and Nordgren [10], Vlasov [11], Simmonds and Danielson [12], Pimenta [13], Ibrahimbegovic [14,15] and Libai and Simmonds [16]. Formulations on nonlinear dynamic shell structures were presented by Simo et al. [17], Kuhl and Ramm [18], Brank et al. [19], Campello et al. [20], among others. Dynamic instability was analyzed by authors like Brank et al. [21] and Delaplace et al. [22]. The numerical time integration recommended therein is probably the simplest way to introduce some energy dissipation in nonlinear dynamic problems, keeping the second order accuracy of the original Newmark algorithm. We would like to compare the examples presented therein with our formulation in future works. In [20], general hyperelastic materials can be used for nonlinear dynamic analysis of shells with rotational degrees-of-freedom.

Recent studies analyze the behavior of advanced shell structures, like foldable structures [23,24] and deployable structures [25,26], structures that can be transported in a compact form and deployed to their full extent when needed, metamaterials [27,28], whose unusual properties derive from their structure, rather from their composition, morphing shell structures [29–31], shells capable of undergoing large changes in shape, whilst remaining within the material's elastic range, and multistable structures [32–34], which have more than one stable state and can move elastically from one state to another.

Following the creased hemisphere presented in [34], this paper presents a dynamic formulation for simulating that hemisphere and other similar shell structures. The presented formulation is geometrically exact for nonlinear applications involving large displacements and large rotations. We emphasize that the main novelty of present work is, contrary to our previous paper [20], the establishment of the weak form for dynamic shell models by using the symmetric Principle of Virtual Work (PVW), together with the

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update description of rotation using Rodrigues parameters. This was motivated by the objective of studying the stability of shell structures, particularly the creased-domes. For that, it is desirable to establish a weak form that can be obtained from a potential. Then, the stability assessment is more direct. We intend to make a deeper study on future works by tracking the evolution of the system natural frequencies. In this context, the Lyapunov stability criterion can be claimed, such as previously done for cable-like structures in [35]. Furthermore, when addressing the PVW to establish the weak form, the static problem becomes a particular case of dynamics, recovering the results from [20,36]. We need that to compare dynamics to statics, as performed in the present work of numerical examples.

The scope of this work is to present a numerical procedure to analyze challenging shell problems (from a numerical point of view). The triangular finite element used, the T6-3i [37], offers great flexibility for unstructured curved meshes generation and presented excellent results. The hemisphere simulations were repeated in LS-Dyna, a commercial finite element package to provide a comparison with the simulations using T6-3i elements.

2. Shell modeling

2.1. Kinematics

The shell model presented here is an extension of the geometrically exact formulation derived in [38]. The finite shell element devised in [38], the T6-3i element, is triangular, allowing robust and versatile numerical discretization. The formulation is pure-displacement based, where no mixed or hybrid types of variables were used, it is free of locking effects due to the incompatibility of the element to the rotations field, and the degrees of freedom used are simple and physically meaningful: the displacements and rotations of the shell director. The kinematics is of the Reissner–Mindlin type, which takes into account the effects of shear deformations.

In this work, the rotation tensor $\boldsymbol{Q}$ is expressed in terms of the Rodrigues rotation parameters, as in [20]. The parameterization with the Rodrigues rotation vector leads to simpler and more efficient expressions compared to the Euler parameterization, as it is totally free of trigonometric functions.

The Rodrigues rotation vector is defined by [39]:

$$\boldsymbol{\alpha} = \frac{\tan(\theta/2)}{\theta/2}\boldsymbol{\theta} \tag{1}$$

where $\boldsymbol{\theta}$ is the classical Euler rotation vector representing an arbitrary finite rotation on 3D space and $\theta = \|\boldsymbol{\theta}\|$ is its magnitude —the Euler rotation angle. The rotation tensor $\boldsymbol{Q}$ expressed in terms of the Rodrigues rotation parameter $\boldsymbol{\alpha}$ can therefore be written as [38]:

$$\boldsymbol{Q} = \boldsymbol{I} + \frac{4}{4+\alpha^2}\left(\boldsymbol{A} + \frac{1}{2}\boldsymbol{A}^2\right) \tag{2}$$

with $\alpha = \|\boldsymbol{\alpha}\|$, and $\boldsymbol{A} = skew(\boldsymbol{\alpha})$.[1]

The angular velocity operator $\boldsymbol{\Omega} = \dot{\boldsymbol{Q}}\boldsymbol{Q}^T$ is the skew-symmetric spin tensor associated to the rotation $\boldsymbol{Q}$. Its axial vector $\boldsymbol{\omega} = axial(\boldsymbol{\Omega})$ is the spin vector or angular velocity vector. Using Rodrigues parameters one can obtain:

$$\boldsymbol{\omega} = \boldsymbol{\Xi}\dot{\boldsymbol{\alpha}} \tag{3}$$

where the tensor $\boldsymbol{\Xi}$ relates $\boldsymbol{\omega}$ to the time derivative of $\boldsymbol{\alpha}$ and is given by:

$$\boldsymbol{\Xi} = \frac{4}{4+\alpha^2}\left(\boldsymbol{I} + \frac{1}{2}\boldsymbol{A}\right) \tag{4}$$

The back-rotated counterpart of $\boldsymbol{\omega}$ can be obtained by $\boldsymbol{\omega}^r = \boldsymbol{Q}^T\boldsymbol{\omega} = \boldsymbol{\Xi}^T\dot{\boldsymbol{\alpha}}$, where the notation with a superscript "r" defines back-rotated quantities. Upon time differentiating $\boldsymbol{\omega}$, one can obtain the angular acceleration vector:

$$\dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\Xi}}\dot{\boldsymbol{\alpha}} + \boldsymbol{\Xi}\ddot{\boldsymbol{\alpha}} \tag{5}$$

with

$$\dot{\boldsymbol{\Xi}} = \frac{1}{2}\frac{4}{4+\alpha^2}\left[\dot{\boldsymbol{A}} - (\boldsymbol{\alpha}\cdot\dot{\boldsymbol{\alpha}})\boldsymbol{\Xi}\right] \tag{6}$$

In the parameterization with Rodrigues rotation vector, due to the definition in Eq. (1), the rotation angle must be restricted to $-\pi < \theta < \pi$. However, with an updated formulation, this is not a limitation as rotations may not exceed π within a single time increment. Using an updated-Lagrangian framework, displacements and rotations must be updated after each time-step.

Fig. 1 (adapted from [36]) shows the shell updated model. A plane shell mid-surface is assumed at the initial reference configuration. At this configuration, it is defined a local orthonormal system $\{\boldsymbol{e}_1^r, \boldsymbol{e}_2^r, \boldsymbol{e}_3^r\}$, with corresponding coordinates $\{\xi_1, \xi_2, \zeta\}$. The vectors $\boldsymbol{e}_\alpha^r$ $(\alpha = 1, 2)$ are placed on the shell mid-plane and $\boldsymbol{e}_3^r$ is normal to this plane.

In this reference configuration, the position $\boldsymbol{\xi}$ of any material point can be described by the vector field:

$$\boldsymbol{\xi} = \boldsymbol{\zeta} + \boldsymbol{a}^r \tag{7}$$

where the vector $\boldsymbol{\zeta} = \xi_\alpha \boldsymbol{e}_\alpha^r$ describes the position of points on the reference mid-surface and $\boldsymbol{a}^r = \zeta\boldsymbol{e}_3^r$ is the shell director, with $\zeta \in H = \left[-h^b, h^t\right]$ as the thickness coordinate and $h = h^b + h^t$ as the shell thickness in the reference configuration.

At instant "i", it is defined a local orthonormal system $\{\boldsymbol{e}_1^i, \boldsymbol{e}_2^i, \boldsymbol{e}_3^i\}$, with $\boldsymbol{e}_i^i = \boldsymbol{Q}\boldsymbol{e}_i^r$ (see Fig. 1), with $\boldsymbol{e}_3^i$ aligned with the director at this instant and $\boldsymbol{e}_\alpha^i$ normal to it. Note that the director is not necessarily normal to the deformed mid-surface, thus accounting for first order shear deformations. A general material point in this configuration can be described by:

$$\boldsymbol{x}^i = \boldsymbol{z}^i + \boldsymbol{a}^i \tag{8}$$

where $\boldsymbol{z}^i = \hat{\boldsymbol{z}}(\xi_\alpha)$ is the position of a material point on the middle surface and $\boldsymbol{a}^i$ is the director at this point, obtained by $\boldsymbol{a}^i = \boldsymbol{Q}\boldsymbol{a}^r$.

Similarly, the position of any material point in the configuration at instant "$i+1$", the end of the present time-step, is described by:

$$\boldsymbol{x}^{i+1} = \boldsymbol{z}^{i+1} + \boldsymbol{a}^{i+1} \tag{9}$$

Here $\boldsymbol{a}^{i+1} = \boldsymbol{Q}^\Delta\boldsymbol{a}^i$, where $\boldsymbol{Q}^\Delta$ is the tensor representing the rotation between instants "i" and "$i+1$". The index "Δ" refers to quantities relating the instants "$i+1$" and "i". As $\boldsymbol{a}^i = \zeta\boldsymbol{e}_3^i$, then, $\boldsymbol{x}^{i+1} = \boldsymbol{z}^{i+1} + \zeta\boldsymbol{Q}^\Delta\boldsymbol{e}_3^i$, and time-differentiating this expression, one can obtain the velocity vector of any material point as:

$$\dot{\boldsymbol{x}}^{i+1} = \dot{\boldsymbol{z}}^{i+1} + \zeta\dot{\boldsymbol{Q}}^\Delta\boldsymbol{e}_3^i = \dot{\boldsymbol{z}}^{i+1} + \zeta\boldsymbol{\omega}\times\boldsymbol{e}_3^{i+1} \tag{10}$$

The displacement associated with any point of the middle plane is given by vector $\boldsymbol{u}$, and can be updated by:

$$\boldsymbol{u}^{i+1} = \boldsymbol{u}^i + \boldsymbol{u}^\Delta \tag{11}$$

The rotations can be updated by [20]:

$$\boldsymbol{\alpha}^{i+1} = \frac{4}{4-\boldsymbol{\alpha}^\Delta\cdot\boldsymbol{\alpha}^i}\left(\boldsymbol{\alpha}^\Delta + \boldsymbol{\alpha}^i + \frac{1}{2}\boldsymbol{\alpha}^\Delta\times\boldsymbol{\alpha}^i\right) \tag{12}$$

where $\boldsymbol{\alpha}^\Delta$ is the rotation vector occurred from configuration "i" to "$i+1$".

[1] The $skew(\boldsymbol{v})$ function transforms a vector $\boldsymbol{v} \in \mathcal{V}_3$ in a skew-symmetric tensor $\boldsymbol{V}$, whose axial vector is $\boldsymbol{v}$. The $axial(\boldsymbol{V})$ function transforms $\boldsymbol{V}$ in its axial vector $\boldsymbol{v}$. Let two vectors $\boldsymbol{v}, \boldsymbol{w} \in \mathcal{V}_3$, the cross product of these vectors gives $\boldsymbol{v}\times\boldsymbol{w} = \boldsymbol{V}\boldsymbol{w}$.