Finite Elements in Analysis and Design 108 (2016) 81–95

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Multiscale finite element analysis of wave propagation in periodic solids

F. Casadei [a,b], J.J. Rimoli [a,*], M. Ruzzene [a,c]

[a] School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, United States
[b] School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, United States
[c] School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, United States

ARTICLE INFO

Article history:
Received 2 April 2015
Received in revised form
28 September 2015
Accepted 1 October 2015
Available online 21 October 2015

Keywords:
Geometric multiscale finite element method
Wave propagation
Periodic solids
Dispersion relations

ABSTRACT

This paper reports on the application of the geometric multiscale finite element method for the analysis of wave propagation in heterogeneous periodic solids. The proposed scheme exploits multi-node elements to describe the microstructure through a local, auxiliary mesh that resolves the fine scale features, and that is used to numerically compute a set of interpolation functions employed for elements formulations at the global level. The method is applied for the analysis of the dispersion properties of, and transient wave propagation in domains featuring periodicity in two dimensions. Band diagram calculations, wave velocities and time domain computations are conducted on solids discretized using two-dimensional and three-dimensional multiscale finite element meshes. Results for assemblies with periodic inclusions, phononic stubbed plates and structural lattices illustrate the effectiveness of the method. Accurate predictions of dispersion relations, wave modes and time domain simulations are obtained with significant reductions in model size. The presented examples also illustrate some of the interesting wave characteristics of the considered class of periodic structures, which include wave directionality and frequency bandgaps.

1. Introduction

Extensive research is devoted to the analysis and design of periodic structures and metamaterials for acoustic waves management [1]. Such assemblies exhibit interesting wave propagation properties such as bandgaps [1–3], response directionality [4–6], left-handedness [7], and negative acoustic refraction [8]. All of these features can be employed for the design of acoustic devices operating over a broad range of frequencies and length scales. The application of such concepts can, for example, be used to perform a variety of acoustics-based signal processing functions at frequencies where electronics suffer from severe power limitations. In conjunction to telecommunication and signal processing, potential implications of the "acoustic wave guiding" technology include among others active sensing of structural integrity [9], and dissipation of high frequency modes of vibration [10]. Other novel structural configurations may be exploited for devices which exhibit acoustic super-lensing, super-focusing and/or cloaking characteristics [7,8].

Unusual wave properties of the kind mentioned above are associated with material and structural heterogeneities, which mostly correspond to periodic modulations of stiffness and inertial properties. Such modulations may result from the periodic dispersion of inclusions of different materials within a matrix [1,5], or from the microstructural configuration of a given assembly [6,11]. The spatial scales of property and geometry modulations dictate the range of frequencies at which wave guiding occurs, and therefore determines the scale and frequency range of application of a device. Very often the size of the microstructure is comparable to the wavelength of the propagating wavefield. Hence, the discretization and simulation of such problems may result in extremely high computational costs as the numerical grid must resolve both the cell and the microstructure length scales. The need for a simple and reliable numerical scheme is also motivated by the emerging field of band structure optimization [12,13] where numerous repeated calculations are required to converge toward a desired design configuration.

Many techniques have been developed to predict the relevant properties of heterogeneous structures and materials, without the need to fully resolve the smallest spatial scales. For instance, homogenization based methods have the objective of describing the overall behavior of an heterogeneous system in terms of equivalent properties [14–17]. For example, composite materials are classically analyzed in terms of effective properties based on rules of mixtures [18]. Also, periodic structural lattices have been

* Corresponding author.
E-mail address: rimoli@gatech.edu (J.J. Rimoli).

http://dx.doi.org/10.1016/j.finel.2015.10.002
0168-874X/

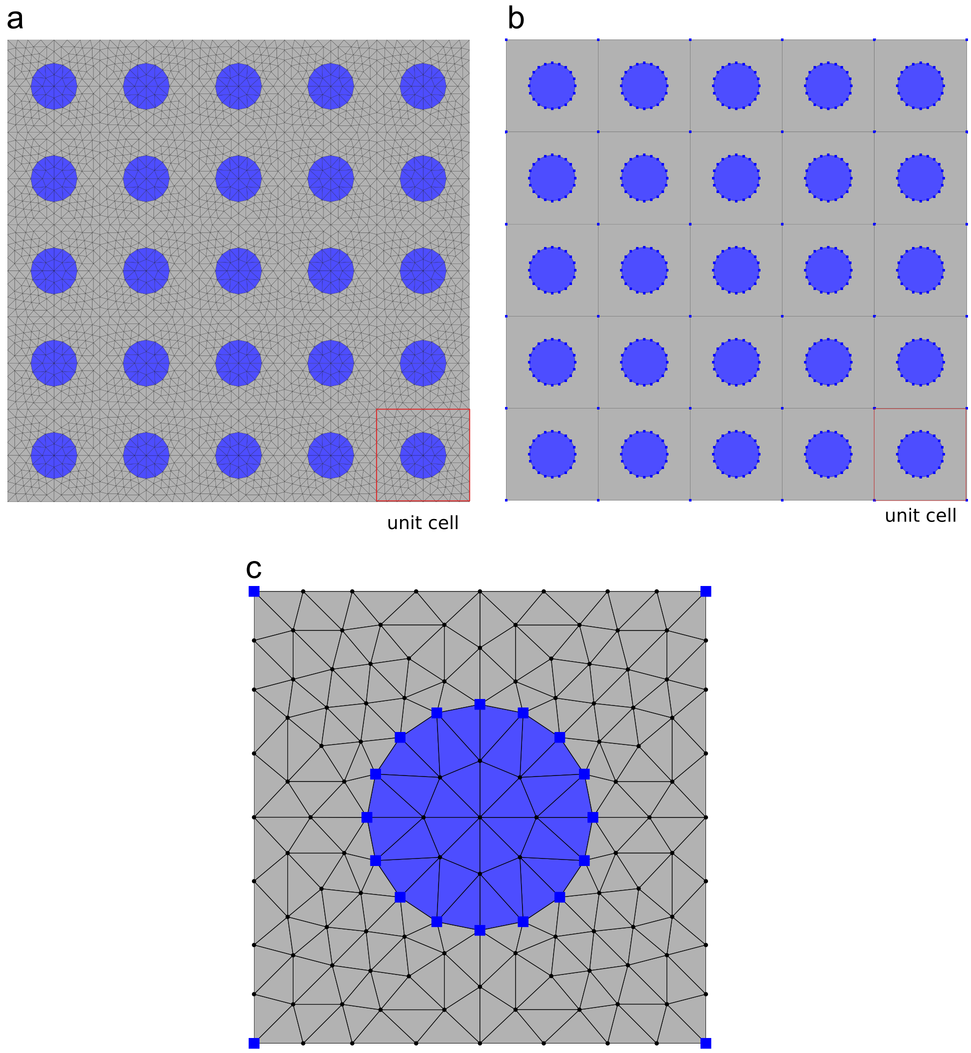

Fig. 1. Periodic assembly with mesh driven by local heterogeneity (a), and MSE-based discretization (b). Detail of MSE of the unit cell with corresponding fine scale triangulation and partition between local (•) and global (■) nodes (c).

studied through homogenization strategies whereby the spatial periodicity is exploited to obtain dynamic equilibrium equations in the Fourier domain [19–23]. Homogenization methods are known to provide accurate results if the macroscopic fields are constant or slowly varying within a single representative volume element. Typically, such requirements hold far from the boundaries of the computational domain and when the size of the microstructure is significantly smaller than the wavelength of deformation.

In order to overcome the limitations of homogenization-based techniques, significant research effort is focused on the development of higher order expansion methods capable of describing the high frequency response and the dispersive behavior of periodic media. For example, Fish et al. [24–29] proposed a multi-grid solver for wave problems in heterogeneous media characterized by microstructures whose size is comparable to that of the structural details or the wavelength of a traveling signal. Similarly, Murakami et al. [30–32] proposed a mixed theory based on high-order continuum models to simulate elastic wave dispersion in heterogeneous composite media. More recently, a reduced Bloch mode expansion method has been introduced by Hussein [33] for the calculation of band diagrams of periodic solids. The method, based on a FE discretization of the unit cell, employs a limited number of Bloch eigenmodes to project the fine-scale eigenvalue problem onto a reduced subspace selected within the irreducible Brillouin zone at high symmetry points. Being in line with the well known concept of modal analysis, the approach maintains accuracy while reducing the computation time even if the calculation of the reduced basis requires explicit solution of the local eigenvalue problem. The idea of using a set of complex Bloch eigenmodes to formulate a specialized multiscale FEM solver is also found in the work by Brandsmeier et al. [34].

Despite the outstanding achievements in the field of high-order homogenization, efficient computation of dispersion and the simulation of wave propagation in heterogeneous media is still a considerable challenge. In this paper, the geometric multiscale finite element method (GMsFEM) recently developed by the authors [35] is applied to the analysis of wave propagation in heterogeneous periodic media. In is worth mentioning that this approach was already shown to be precise and computationally efficient for the analysis of elastic wave scattering from localized defects [36]. The approach is based on the formulation of multiscale elements (MSEs) with an arbitrary number of nodes that are used to model heterogeneities occurring at sub-cell length scales. In contrast to other multiscale strategies [33,34] that require the solution of a complex eigenvalue problem to construct the enrichment basis, the proposed multiscale shape functions are expressed in terms of a simple algebraic operator. In addition, the adopted numerical scheme can be also exploited for the explicit transient simulation of wave propagation in periodic domains of finite size.