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Aero-elastic characteristics of tapered plate wings

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ABSTRACT

In the present work an aero-elastic model is presented to study flutter and divergence of isotropic plate wings. A finite element model is subsequently developed to apply the theoretical model and predict the performance of plate wings. A divergence analysis is carried out using the finite element model combined with the vortex lattice method for aerodynamic load calculations. The flutter analysis is carried out using a finite element model combined with the doublet lattice method. The aerodynamic model is coupled to the structural model using the shape (interpolation) functions of the finite element model. Static condensation is used to express the finite elements' in-plane degrees of freedom in terms of the bending ones, hence reducing the number of the elements' degrees of freedom per node to three. A MATLAB code is developed to implement the proposed model using three-node triangular finite elements. The present model is validated using benchmark problems available in the literature, and the effects of taper ratio on both divergence and flutter speeds and flutter frequency are studied.

1. Introduction

One of the most powerful methods used to study the behavior of structures is the finite element method. The idea of the finite element method is based on discretizing the structure into small elements in which the solution can be approximated using simple interpolation functions. All the elements are assembled together to construct a system of equations relating the loads to the degrees of freedom of the whole structure. Many researchers used the finite element method to study the aero-elastic characteristics of aircraft structures.

Cole [1] developed a model to study the divergence of wings with high aspect ratios. The results were compared with experimental data. The results showed that the divergence speed decreases with the increase of the forward sweep angle of the wing. Lee and Lee [2] performed a vibration study for composite plate wings. The wings considered in the study had different sweep back angles and taper ratios. The effect of fiber orientations was investigated. The finite element model was built using four and eight-node quadrilateral elements. The results were validated by comparison to experimental results. Shokrieh and Behrooz [3], studied the dynamic instability of a wing using NISA finite element program [4]. The purpose of this work was to ensure that the wing was safe from flutter within the wing speed range based on the Joint Aviation requirements (JAR-23) [5]. Koo [6] performed static and dynamic aero-elastic analyses for isotropic plate wings with different sweep back angles. Furthermore, he studied the effect of different double-sweep wing angles on flutter and divergence. A finite element model was used together with the doublet lattice method using the surface spline method. The effect of different fiber orientations was demonstrated. Moosavi [7] introduced a model to calculate the flutter speed for wings using Galerkin's method. The model used the finite element method in the structural analysis and the strip theory in the aerodynamic analysis. Su and Cesnik [8] examined a flying wing aero-elastic characteristics using the finite element method. The finite element model was built using a non-linear beam model, and the 2D finite state inflow theory was used in the aerodynamic model.

Hollowell and Dugundji [9] demonstrated experimental and analytical analyses for a composite plate wing. An experimental investigation was carried out for a cantilever plate wing with different fiber orientations. The results of the analytical analysis were compared with the experimental ones and showed good agreement. An instability analysis was carried out using the V-g method [10]. Kameyama and Fukunaga [11] introduced an analysis and design for a composite plate wing using the genetic algorithm. The finite element method was used to obtain the structure stiffness matrix, and the doublet lattice method was used for the aerodynamic analysis. The instability analysis was carried out using the p–k method [12]. The results were compared with the experimental data obtained in Hollowell and Dugundji [9]. Moon [13] used multidisciplinary optimization to improve the wing divergence speed using the vortex lattice to derive the aerodynamic coefficient matrix and the finite element method to obtain the wing

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List of symbols

Symbol	Definition
$\mathbf{A_{Dlm}}$	unsteady aerodynamic coefficient matrix
A_{panel}	aerodynamic panel area
$\mathbf{A_{sd}}$	unsteady aerodynamic coefficient matrix in structural coordinates
$\mathbf{A_{ss}}$	steady aerodynamic coefficient matrix in structural coordinates
$\mathbf{A_{vlm}}$	steady aerodynamic coefficient matrix at the aerodynamic panels
$\mathbf{B}$	strain–displacement matrix
$\mathbf{D}$	stress–strain matrix
dy	aerodynamic panel length along span
$\mathbf{F}$	load vector
$\mathbf{K}$	stiffness matrix
$\mathbf{K^e}$	element stiffness matrix in local coordinates
L_1, L_2, L_3	element area coordinates
$\mathbf{M}$	mass matrix
$\mathbf{M^e}$	element mass matrix in local coordinates
$\mathbf{N}$	shape function matrix
$\mathbf{q}$	displacement vector
$\mathbf{w_s}$	structural bending displacement vector
$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$	element local coordinates
$\mathbf{X, Y, Z}$	structural global coordinates
ζ	wing damping ratio
m	mass
Λ^e	strain energy
E	elasticity modulus of the wing material
V	volume
k	reduced frequency
$\mathbf{GN_1}$	shape function numerical integration matrix
G	shear modulus
$\mathbf{GN_{2d}}$	interpolation matrix using shape function for divergence analysis
$\mathbf{Bc}$	flutter boundary condition matrix
ρ	air density
KE	kinetic energy
ω	flutter frequency
t	plate wing thickness
$\mathbf{b}$	reference length (half the wing root chord)
σ	stress
$\mathbf{GN_{2f}}$	interpolation matrix using shape function for flutter analysis
U_∞	flow speed
q_∞	dynamic pressure
ε	strain

stiffness matrix. The wing box was approximated by an equivalent flat plate. The design objective was to minimize the aircraft weight under divergence and flutter speeds constraints. The proposed optimization model resulted in a significant improvement in the wing flutter speed. Recently, Leon [14] used aero-elastic tailoring to improve the flutter speed of a composite plate wing using topology optimization. The aero-dynamic analysis was made using the doublet lattice method, and the structural analysis was made using the finite element method. The Spline method was used to connect between the aerodynamic model and the finite element model. The V–g method was used to calculate the flutter speed. The aero-elastic stability analysis was performed using Zaero software package [15].

Patil and Hodges [16] presented a theory for the aero-elastic analysis of highly flexible flying wing aircrafts. The structural analysis is performed using beam model while the aerodynamic analysis is obtained using the 2-D aerodynamic theories. The solution of the aero-elastic equation was obtained using Newton–Raphson method. A nonlinear simulation was obtained for the flying wing aircraft. Kennedy and Martins in 2014 [17] a 3-D aero-structural optimization model. The aerodynamic analysis is obtained using a 3-D panel code while the structural analysis is performed using a finite element code. They developed a general model to connect between the aerodynamic model and the finite element model. The parallel Newton–Krylov method was used to sole the aero-structural equations. They performed an aero-structural optimization for a real wing.

The aim of this work is to study the effect of wing taper ratio on the static and dynamic instabilities of the wing. Complete structural and aerodynamic models are developed and coupled together to perform the required instability analyses. A MATLAB code is developed for the aerodynamic, finite element, and aero-elastic analysis which uses the shape function transformation to couple between the aerodynamic and the finite element models.

2. Aero-elastic model

Two Aero-elastic models are developed to study the interaction between inertial, elastic and aerodynamic forces. There are two types of aero-elastic phenomena: The first one represents the interaction between steady aerodynamic forces and static structural elastic forces, which is known as static aero-elasticity, and the second one represents the interaction between the unsteady aerodynamic forces and the structural elastic and inertial forces, which is known as dynamic aero-elasticity (flutter).

2.1. Aerodynamic analysis

In the aerodynamic analysis the steady and the unsteady aerodynamic coefficient matrices are developed.

2.1.1. Steady aerodynamic analysis

There are various methods used to determine the aerodynamic forces on lifting surfaces. One of the simplest and most accurate methods is the **Vortex Lattice Method** (VLM). In this section a steady aerodynamic model is developed for the calculation of the wing static loads needed for divergence analysis.

In the **VLM**, the plan-form of the wing is divided into panels (elements). Each panel is replaced by a horseshoe vortex. This horseshoe vortex has a vortex filament across the quarter-chord of the panel and two filaments stream-wise, one on each side of the panel starting at the quarter-chord and trailing downstream in the free-stream direction to infinity. Fig. 1 shows a typical horseshoe-vortex representation of a wing plan-form. The boundary condition for each horseshoe vortex is satisfied by requiring the inclination of the fluid streamline to match the angle of attack at the three-quarter-chord point of its elemental panel. The circulations required to satisfy this tangent flow boundary condition are then determined by solving a matrix equation.

$$\mathbf{w_m} = -U_\infty \boldsymbol{\alpha} \qquad (1)$$

where $\boldsymbol{\alpha}$ is the angle of attack at the control points of the aerodynamic mesh.

We can calculate the Influence coefficient matrix by calculating the down-wash velocity at each control point by determining the relative influence of the singularities of a planar wing based on