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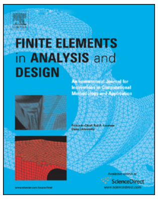

Modelling the prestress transfer in pre-tensioned concrete elements

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ABSTRACT

Three models were developed to simulate the transfer of prestress force from steel to concrete in pre-tensioned concrete elements. The first is an analytical model based on the thick-walled cylinder theory and considers linear material properties for both steel and concrete. The second is an axi-symmetric finite element (FE) model with linear material properties; it is used to verify the analytical model. The third model is a three dimensional nonlinear FE model. This model considers the post-cracking behaviour of concrete as well as concrete shrinkage and the time of prestress releasing. A new expression from the analytical model is developed to estimate the transmission length as well as the stress distribution along the tendon. The paper also presents a parametric study to illustrate the impact of diameter of prestressing steel, concrete cover, concrete strength, initial prestress, section size, surface roughness of prestressing steel, time of prestress release, and the member length on the transfer of stress in pre-tensioned concrete elements.

1. Introduction

The prestress force in pre-tensioned concrete elements is transferred from the steel to the concrete over a certain length, which is known as the transmission length. The formulae of the transmission length in the current design code are basically developed empirically for normal types of concrete. Because of the rapid innovation in construction industry and introducing of new types of concrete materials (i.e. high-strength, self-compacting etc.), experimental tests are needed to estimate the transmission length [1]. Models account for the new type of concrete material, based on the mechanical properties of materials which can be measured using simple tests, will be desirable [2].

The transmission length is influenced by many factors such as diameter of prestressing steel, initial or effective prestress, concrete strength, type of release, type of tendon, bond condition, concrete cover, surface condition and size of the section on the transmission length [3–11]. To date, no full agreement exists on the factors used in formulae to predict the transmission length [3].

The aim of this study is to develop a model that accounts for different concrete materials and reinforcing steel as a closed-form expression to predict the transmission length to be used in initial design stage where new concretes are used. Moreover, this study will develop a finite element model to understand the influence of different parameters on the prestress transfer.

This paper is organised in nine sections. Sections 2 and 3 give a brief background about the previous modelling work and the transfer of prestress, respectively. In Section 4, an analytical expression to calculate the transmission length and the stress distribution is given. Axi-symmetric and 3D finite element (FE) models are developed in Sections 5 and 6. Section 7 presents a parametric study while Section 8 examines the assumptions used in the analytical model. Finally, a summary and conclusions are given in Section 9.

2. Background

Modelling the transfer of prestress force in pre-tensioned concrete elements has been described either by using empirical or numerical models (i.e. analytical and finite element (FE) models). These models allow for calculating the transmission length that is needed to transfer the prestress force in steel to concrete.

2.1. Analytical modelling background

Analytical modelling of prestress transfer was previously carried out by considering the prestressing steel as a solid cylinder and concrete as a hollow cylinder with inner radius equal to the prestressing steel radius and with an infinite outer radius [12]. Although the model assumed an infinite radius for concrete and

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Nomenclature

A_c	cross sectional area of concrete
A_p	cross sectional area of prestressing steel
a	internal radius of cylinder
b	external radius of cylinder
d	the nominal diameter of the prestressing
dx	length of small element in longitudinal direction
E	Young's modulus
E_c	concrete Young's modulus
E_p	prestressing steel Young's modulus
e	eccentricity, which defines the rate at which the function approaches the asymptote
f_{cx}	average longitudinal stress in concrete at x
f_{cu}	concrete compressive strength after 28 days in MPa
f_{px}	longitudinal stress in prestressing steel at x
f_{bx}	bond stress at x
f_{pe}	effective prestress (beyond transmission zone)
f_{pi}	initial prestress
f_t	tensile strength
G	Drucker–Prager hyperbolic function of flow potential
G_f	fracture energy
K_c	ratio of the second invariant on the tensile meridian to that on compressive meridian
l_t	transmission length
n_p	number of mesh segments around the tendon
p	radial pressure
p_i	applied internal pressure in the radial direction
p_e	applied external pressure in the radial direction
$\overline{p}$	hydrostatic pressure
$\overline{q}$	von Mises equivalent stress
r	deformed nominal radius of the prestressing strands or bars
r_p	nominal radius of prestressing steel bar
$r_{c,1}$	internal radius of concrete cylinder which equals radius of steel bar after prestressing
$r_{c,2}$	external radius of concrete cylinder
t	age in days
T	degree of temperature
u_r	radial deformation at a radius equal r
$u_r{}^p$	radial deformation of pre-stressing bar at the outer perimeter
$u_r{}^c$	radial deformation of concrete cylinder at the inner perimeter
x	position along the transmission length
α_T	coefficient of thermal expansion
ΔT	change in temperature
ε_{sh}	shrinkage strain
ν	Poisson's ratio
ν_c	Poisson's ratio of concrete
ν_s	Poisson's ratio of steel
σ_{b0}	initial equibiaxial compressive yield stress
σ_{c0}	uni-axial compressive yield stress
$\hat{\overline{\sigma}}_{max}$	maximum principal effective stress
σ_{t0}	uni-axial tensile stress at failure
σ_z	applied stress in the longitudinal direction
τ	tangential stress
μ	coefficient of friction
ψ	angle of dilation

neglected the effect of longitudinal stress on concrete, a simple expression for prestress transfer and transmission length was given. The thick-wall cylinder model was also used to evaluate the effects of concrete cover on bond behaviour of prestressing strands in both high and normal concrete strengths considering the non-linear behaviour of concrete in tension [8]. The same concept was used to estimate the transmission length in pre-tensioned concrete element [9]. The model by Oh et al. [9] considered non-linear anisotropic concrete behaviour after the occurrence of cracks and assumed no slip. The slip of prestressing steel was considered in evaluation of the transmission length in the study of Benítez and Gálvez [1].

The literature shows that the use of the thick-wall cylinder concept in modelling of prestress transfer is simple and provides a more rational basis. On the other hand, the reviewed literature did not present a closed-form mathematical expression to estimate the transmission length and stress distribution along prestressing steel.

2.2. Finite element modelling background

The analytical modelling becomes very complicated when the material's non-linearity and the effect of different types of stresses in addition to the behaviour in 3D are taken into account. The FE method on the other hand is much more effective in handling the complexities associated with material non-linearity and the structural behaviour in 3D.

The FE method has been used by other researchers to study the effects of the releasing techniques of prestressing steel on the stress field and cracks at the end zone [13]. Kannel et al. [13] used the ABAQUS 5.4 software to model pre-tensioned concrete girders using three dimensional continuum elements for concrete and truss elements for the strands. The transfer of the prestress force from steel to concrete was modelled by varying the strand diameter linearly from zero at the end of the girder to the nominal diameter at the end of the transmission length. Another method was also used by Kannel et al. [13] in which the interactions between the steel strand and concrete were modelled by using connected rigid springs with plastic behaviour. The use of truss elements, which account only for axial forces, neglects the effect of radial deformation due to Poisson's effect. These techniques and the assumption of a linear material model for both steel and concrete do not accurately reflect the reality of the bond behaviour between prestressing steel and concrete in prestressed concrete elements.

The ANSYS FE package was also used to model pre-tensioned concrete beams and railway sleepers [14,15]. The concrete was modelled using 8-node solid elements and truss elements were used to model the prestressing steel. Concrete cracking and crushing were modelled using the concrete damage plasticity model proposed by Willam and Warnke [16]. The prestress was simulated by assigning initial strain for the truss elements while the interaction between steel and concrete surface was assumed to be fully bonded. The assumption of full bond condition affects the estimation of transfer of prestress force to concrete because it eliminates the contribution of prestressing steel slippage.

In the work of Ayoub and Filippou [17], the prestressing process was modelled across the different stages in the pre-tensioned concrete elements (i.e. prestressing, casting and releasing) using FE models in which an empirical local bond relationship was used. The main shortcoming of this model and the models before is the neglecting of the concrete tension softening behaviour.

Recently two FE approaches were developed to model the pre-tensioned concrete element, namely: the embedment approach