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Numerical analysis of elastic wave propagation in unbounded structures

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ABSTRACT

The main objective of this paper is to show the effectiveness and usefulness of the concept of an absorbing layer with increasing damping (ALID) in numerical investigations of elastic wave propagation in unbounded engineering structures. This has been achieved by the authors by a careful investigation of three different types of structures characterised by gradually increasing geometrical and mathematical description complexities. The analysis includes propagation of longitudinal elastic waves in a 1-D semi-infinite isotropic rod, modelled according to the classical 1-mode theory of rods, propagation of coupled shear and flexural elastic waves in a 1-D semi-infinite isotropic beam modelled according to the Timoshenko beam theory, as well as propagation of elastic waves in a 3-D semi-infinite isotropic half-pipe shell modelled by a 6-mode theory of shells. The concept of the ALID has been not only presented by the authors, but certain relations between the ALID properties and the characteristics of propagating elastic waves have been given that can help to maximise the ALID performance in terms of its damping capability. All results of numerical calculations presented by the authors in this work have been obtained by the use of the Time-domain Spectral Finite Element Method (TD-SFEM).

1. Introduction

Recently various structural health monitoring (SHM) techniques have become the subject of extensive scientific investigations [1]. Among many techniques used for that purpose those based on elastic wave propagation and wave scattering have become widely exploited both experimentally [2] and numerically [3]. However, in many cases numerical investigations are strongly influenced by scale factors resulting from the fact that the lengths of propagating elastic waves are very often much smaller than characteristic structural dimensions. This usually leads either to large numerical models of millions degrees of freedom in the first case [4,5] or unwanted boundary effects [6,7] in the second case, when only selected parts of structures are investigated. In that context numerical techniques enabling one not only to reduce the boundary effects, but also to reduce the size of numerical models, are strongly sought and required.

It should be noted that unwanted wave reflections from boundaries may influence or mask reflections resulting from the presence of structural defects making structural analysis very complex. In the case of numerical models employed to solve various wave propagation problems removal of unwanted boundary reflections is equivalent to representing total radiation outside the area of the study. This problem can be solved by using different methods such as infinite elements [8], boundary integral methods [9], non-reflecting boundary conditions [10], as well as absorbing layer techniques [11].

Infinite element methods are based on special types of elements with modified properties that are aggregated by standard finite element routines, but used to model the infinite space. The application of infinite elements leads to good results in the case of various static problems, as well as in certain cases of wave propagation problems, these being electromagnetism and acoustics. However, results presented in [12–18] prove that infinite elements are not suitable for a high accuracy removal of unwanted boundary reflections in the case of propagation of guided or bulk waves. Also the area of analysis must be much bigger than the area of investigation, which results in an increase in the number of model degrees of freedom.

On the other hand non-reflecting boundary conditions are in fact special types of boundary conditions used in order to model wave propagation in unbounded media [19]. These techniques are based on extra variables used to approximate the infinite dimensions of the media and can be successfully applied in the case of the Finite Element Method (FEM) or the Finite Difference Method

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(FDM). The model dimensions remain the same as the area under consideration. These techniques also lead to good results, but they require certain modifications of standard solution procedures applied by the FEM or the FDM.

The technique of absorbing layers allows the absorption of waves that enter the layers. Certain small reflections from the absorbing layers may exist, but their amplitudes can be reduced by correct definition of layer parameters. Two techniques based on the concept of absorbing layers exist in the literature, known as a perfectly matched layer (PML) or an absorbing layer with increasing damping (ALID). Originally the PML was developed and employed in the case of electromagnetic waves [20,21], but later was extended onto the fields of acoustics [22], seismology [23,24], as well as onto elastic waves [25–28]. Theoretically waves enter the PML without reflections and decay inside exponentially. In practice due to various model discontinuities, mainly arising and present due to numerical reasons between the area under investigation and the layer, small reflections can be observed. For this reason a correct definition of PML parameters, like its length and attenuation, remains essential in order to achieve a proper and efficient model that leads to good results. On contrary to this the ALID utilises the concept proposed in [19]. In this case the absorbing layer is presented by material with its damping properties increasing along the depth of the layer. This method was successfully applied for modelling wave propagation in water [29] and porous media [30].

The main objective of this paper is to show the effectiveness and usefulness of the concept of an absorbing layer with increasing damping (ALID) in numerical investigations of elastic wave propagation in unbounded engineering structures. This has been achieved by the authors by a careful investigation of three different types of structures characterised by their gradually increasing geometrical and mathematical description complexities. The analysis included propagation of longitudinal elastic waves in a 1-D semi-infinite isotropic rod, modelled according to the classical 1-mode theory of rods [31], propagation of coupled shear and flexural elastic waves in a 1-D semi-infinite isotropic beam, modelled according to the Timoshenko beam theory [32], as well as propagation of elastic waves in a 3-D semi-infinite isotropic half-pipe shell modelled by a 6-mode theory of shells [33]. The concept of the ALID has not only been presented by the authors, but certain relations between the ALID properties and the characteristics of propagating elastic waves have been given that can help to maximise the ALID performance in terms of its damping capability. All results of numerical calculations presented by the authors in this work have been obtained by the use of the Timed-domain Spectral Finite Element Method (TD-SFEM) [3].

2. Absorbing layer with increasing damping

The concept of an absorbing layer with increasing damping (ALID) is well described in [13], however it should be mentioned at this point that this idea dates back to 1980s [28]. This concept can be explained by considering a simple 1-D equation of motion in the time domain, written for the layer using the FEM convention [34], as

$$[\mathbf{M}]\{\ddot{q}\}+[\mathbf{C}]\{\dot{q}\}+[\mathbf{K}]\{q\}=\{F\} \tag{1}$$

where $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ are the characteristic inertia, damping and stiffness matrices, respectively, while $\{q\}$ and $\{F\}$ are, respectively, the vectors of nodal displacements and forces dependent on the x co-ordinate only. The symbols $\dot{\square}=d/dt$ and $\ddot{\square}=d^2/dt^2$ denote the first and second time derivatives, respectively.

Under assumption that the damping matrix $[\mathbf{C}]$ within the ALID is a linear combination of both the inertia $[\mathbf{M}]$ and the stiffness $[\mathbf{K}]$ matrices, as well as that harmonic waves can propagate only along the x-axis, it can be written as

$$[\mathbf{C}]=a(x)[\mathbf{M}]+b(x)[\mathbf{K}],\quad \{q\}=\{\hat{q}\}e^{-i\omega t}e^{ikx} \tag{2}$$

where ω and k are the angular frequency and the wave number, respectively, while $a(x)$ and $b(x)$ are certain smooth scaling functions that vary along the depth of the ALID in the following manner:

$$a(0)=b(0)=0,\quad a(l)=b(l)=1 \tag{3}$$

where $x=0$ corresponds to the structure-layer interface and $x=l$ to the full length of the layer. The symbol $i=\sqrt{-1}$ denotes the imaginary unit, while $\{\hat{q}\}$ is the vector of nodal displacement amplitudes.

After substitution of relations (2) into (1) and necessary rearrangement of terms the original equation of motion in the time domain (1) can be represented in the frequency domain as

$$-\rho\left(1+i\frac{a(x)}{\omega}\right)[\tilde{\mathbf{M}}]\omega^2\{q\}+E(1-i\omega b(x))[\tilde{\mathbf{K}}]\{q\}=\{F\} \tag{4}$$

with $[\mathbf{M}]=\rho[\tilde{\mathbf{M}}]$ and $[\mathbf{K}]=E[\tilde{\mathbf{K}}]$, and where ρ and E are the frequency independent material density and elastic modulus, respectively.

From the equation of motion (4) it arises that both density ρ and elastic modulus E can be considered as frequency dependent within the ALID:

$$\rho(\omega)=\rho\left(1+i\frac{a(x)}{\omega}\right),\quad E(\omega)=E(1-i\omega b(x)) \tag{5}$$

which allows us to express the frequency dependent wave number $k(\omega)$ as

$$k^2(\omega)=\omega^2\frac{\rho(\omega)}{E(\omega)}=\omega^2\frac{\rho}{E}(c+id) \tag{6}$$

where

$$c=\frac{1-a(x)b(x)}{1+b^2(x)\omega^2},\quad d=\frac{a(x)+b(x)\omega}{\omega+b^2(x)\omega^3} \tag{7}$$

Based on relations (7) it can be noted that the wave number $k(\omega)$ is complex with its real and imaginary parts remaining positive in the case of elastic waves propagating within the ALID in the positive direction [13]. All such waves are attenuated and their wave numbers vary over the length of the layer.

It should be mentioned here that the part of the damping matrix $[\mathbf{C}]$ proportional to the stiffness matrix $b(x)[\mathbf{K}]$ strongly affects numerical solving of the equation of motion (1). In a general case of the TD-SFEM and problems related with propagation of elastic waves the explicit scheme of central differences is commonly used [3], as the scheme can take full advantage of the diagonal (1-D or 2-D problems) or semi-diagonal (3-D problems) forms of the characteristic inertia $[\mathbf{M}]$ and preferably damping $[\mathbf{C}]$ matrices. However, the part of the damping matrix $b(x)[\mathbf{K}]$ is consistent or full and cannot be effectively diagonalised in this case. Moreover, it also strongly affects the stability of the central difference scheme significantly increasing its minimal time step. On the other hand the part of the damping matrix $[\mathbf{C}]$ proportional to the inertia matrix, i.e. $a(x)[\mathbf{M}]$, is practically free of these drawbacks. For those reasons the damping matrix $[\mathbf{C}]$ is usually assumed in the form

$$[\mathbf{C}]=a(x)[\mathbf{M}],\quad b(x)=0 \tag{8}$$

It can be further assumed that the functions $a(x)$ can be expressed as

$$a(x)=10^{\alpha}x^{\beta},\quad \alpha,\beta>0 \tag{9}$$