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Time-duration extended Hilbert transform superposition for the reliable time domain analysis of five-layered damped sandwich beams

S.H. Bae [a], J.R. Cho [a,b,*], W.B. Jeong [a]

[a] School of Mechanical Engineering, Pusan National University, Busan 609-735, Republic of Korea
[b] Research & Development Institute of Midas IT, Gyeonggi 463-400, Republic of Korea

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ABSTRACT

The direct time domain analysis of five-layered damped sandwich beams subject to impulse load using the discrete Hilbert transform may lead to the unstable growth in damped time responses owing to the incorrectness of Hilbert-transformed imaginary impulse force and the insufficient time duration of applied impulse force. To resolve this problem, a time domain analysis method making use of the Hilbert transform superposition and the time duration extension is introduced in this paper. The incorrect imaginary impulse force near the end of time period, which is caused by the fact that the discrete Hilbert transform using Fourier transform considers the impulse force as a periodic function, is resolved by dividing the external impulse force into a finite number of rectangular impulses. And, the sufficient time duration of applied impulse force is estimated by a Newton–Raphson-based iterative residual method. The validity of the present method is justified from the numerical experiment for analyzing the time response of five-layered damped sandwich beam.

1. Introduction

The damped vibration responses of complex structural dynamics such as sandwich beam structures with viscoelastic layers [1,2] are usually analyzed in either frequency or time domain. In the frequency domain analysis, the external excitation force is expressed as harmonic force so that the imaginary part is automatically defined. But, in the time domain analysis, the external excitation force is real contrary to the complex-valued dynamic equation system. In order to maintain the consistency in the complex-valued dynamic equation system, the real-valued external impulse force can be converted into an analytic force signal by defining the imaginary force signal using Hilbert transform [3–5]. Meanwhile, a state-space formulation in the modal superposition approach to solve the time response of the damped dynamic system leads to two poles which are radial symmetry in the complex plane [3]. The radial symmetry implies that one pole is stable but the other is unstable such that the standard numerical solving techniques cannot provide a successful solution because the unstable pole causes unbounded growth in the damped dynamic response. To avoid such an unbounded growth, the time-reversal technique [6,7] is used, in which the time differential equation corresponding to the unstable pole is converted to one running backwards in time.

However, the time domain analysis making use of the Hilbert transform and the time-reversal technique may lead to the incorrect damped time response owing to two problems, the incorrect Hilbert-transformed imaginary force and the insufficient time duration of applied external force. The Hilbert transform can be analytically derived according to its mathematical definition if the input force signal is expressed in a well-defined explicit function of time. However, for arbitrary input force signals, their Hilbert transforms are alternatively obtained by a combined use of Fourier (FT) and inverse Fourier transforms (IFT) according to the fundamental properties of the complex-valued strong analytical signal [5]. But, such a discrete Hilbert transform using FT and IFT may provide us the incorrect imaginary function near the end of time period because Fourier and inverse Fourier transforms consider the input force signal as a periodic function. Meanwhile, the time duration for the time domain analysis is set based on the original real-valued external force, but it may be insufficient for the imaginary external force. The Hilbert-transformed imaginary force may not be zero at the beginning even though its original real-valued external force starts with zero value, which gives rise to the incorrect unstable time response. This problem is solely owing to the fact that the time duration of complex-valued analytic force is set based on the original real-valued external

* Corresponding author at: School of Mechanical Engineering, Pusan National University, Busan 609-735, Republic of Korea. Tel.: +82 51 510 3088; fax: +82 51 514 7640.
E-mail address: jrcho@pusan.ac.kr (J.R. Cho).

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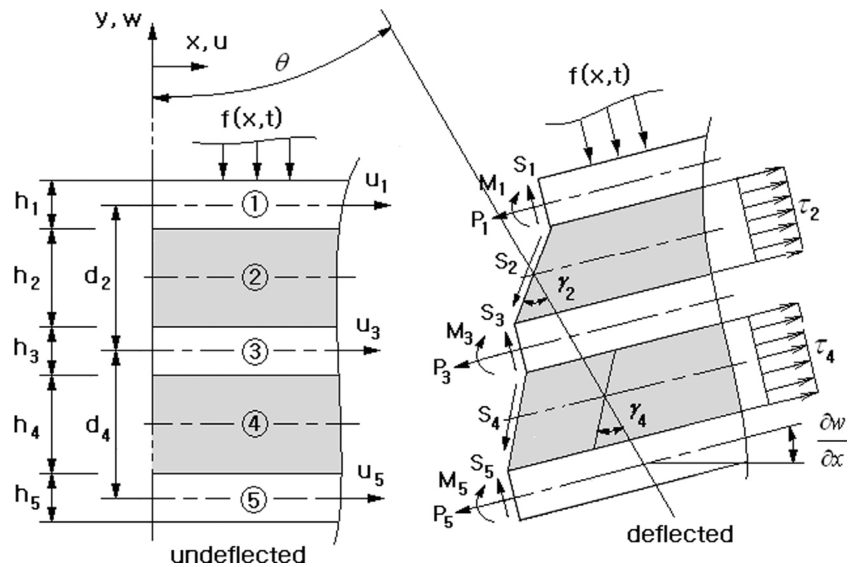

Fig. 1. Strains, forces and moments on the deformed five-layered damped sandwich beam subject to vertical dynamic load $f(x,t)$ (darkened: viscoelastic layers, white: metal layers).

force. It can be resolved by extending the time duration for the time domain analysis so that the imaginary force starts with a sufficiently small value.

As an extension of our work on the forced vibration of damped sandwich beams [8,9], the purpose of this paper is to introduce a time-duration extended Hilbert transform superposition method for the reliable impulse response analysis of five-layered damped sandwich beams in time domain. In order to resolve the above mentioned problem of the discrete Hilbert transform by a combined use of FT and IFT near the end of time period, the Hilbert transform of arbitrary external force is obtained by the superposition method. An arbitrary continuous real-valued force signal is divided into a finite number of rectangular impulses and its Hilbert transform is obtained by summing up the analytic Hilbert transforms of each rectangular impulse. Since the analytic Hilbert transform of rectangular impulse is derived by the mathematical definition of Hilbert transform, the above-mentioned discrepancy near the end of time period can completely disappear. Meanwhile, the original time duration set based on the real-valued external force is extended by a Newton–Raphson-based iteration method, based on the fact that Hilbert transform conserves the magnitude of an original input signal [5]. The difference in squared areas between the original and Hilbert-transformed functions is defined as a residual functional, and the original time duration is successively extended in both the negative and positive time directions until the relative residual satisfies the preset tolerance.

The present method is applied to analyze the time response of five-layered damped sandwich beam [10–12] in order to illustrate and validate the theoretical work. The five-layered damped sandwich beam is modeled using 2-node 8-DOF damped beam elements which are developed based on the principle of virtual work and the compatibility relation between the lateral displacement and the shear strains [9]. The complex finite element matrix equations are derived in both time and frequency domain, and the time-reversal technique is applied to the corresponding state-space formulation in the mode superposition approach. Through the numerical experiments, the validity of the Hilbert transform superposition and the time duration extension algorithm are examined, and the forced time response of five-layered damped sandwich beam is compared with those obtained by IFT and the Hilbert transform superposition method without extending the time duration.

This paper is organized as follows. The finite element approximation of the forced vibration of five-layered damped sandwich beam using 8-DOF damped beam elements is addressed in Section 2. The application of time-reversal technique for the time domain analysis of linear complex structural dynamics is briefly summarized in Section 3. The Hilbert transform superposition for defining the correct imaginary impulse force and the time-duration extension using a Newton–Raphson-based residual method are explained in Section 4. The numerical experiments for illustrating the present method are represented in Section 5 and the conclusion is made in Section 6.

2. Forced vibration of five-layered damped sandwich beam

In the current study, we consider a five-layered damped sandwich beam shown in Fig. 1 as an engineering example of complex structural dynamic problems. The beam of length L and width b with rectangular cross-section undergoes the lateral dynamic deflection under the action of time-varying vertical load $f(x,t)$. Three metal layers of thicknesses h_1, h_3 and h_5 are linear elastic with Young's moduli of E_1, E_3 and E_5, while two darkened layers of thicknesses h_2 and h_4 are linearly viscoelastic with complex shear moduli $G_2^* = G_2(1+i\eta_2)$ and $G_4^* = G_4(1+i\eta_4)$ respectively. Here, η_2 and η_4 indicate the loss factors and $(\cdot)^*$ denotes the complex value throughout this paper. It is assumed that the beam thickness is sufficiently small compared to the beam length and five layers are completely bonded so as not to allow slipping at the layer interfaces. Further assumptions are made for the analysis of the beam transverse displacement: (1) the transverse direct strains in all the five layers are small enough so that the lateral displacement is uniform across any section of the beam, (2) the longitudinal and rotary effects are negligible, (3) three metal layers obey Kirchhoff hypothesis so as not to produce transverse shear strains, and (4) two viscoelastic layers obey Kerwin assumption [13,14] so that the longitudinal direct strains are much small than the shear strains.

The beam can be either symmetric or asymmetric, and the transverse shear strains γ_2 and γ_4 in two viscoelastic layers are calculated by

$$\gamma_I = \frac{d_I}{h_I}\frac{\partial w(x,t)}{\partial x} + \frac{u_{I-1}(x) - u_{I+1}(x)}{h_I}, \quad I = 2, 4 \tag{1}$$