Finite Elements in Analysis and Design 90 (2014) 50–60

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Using a second gradient model to simulate the behaviour of concrete structural elements

Gwendal Jouan [a,b], Panagiotis Kotronis [a,*], Frédéric Collin [b]

[a] LUNAM Université, Ecole Centrale de Nantes, Université de Nantes, CNRS UMR 6183, GeM (Institut de Recherche en Génie Civil et Mécanique), 1 rue de la Noë, BP 92101, 44321 Nantes Cedex 3, France

[b] Département Argenco, Université de Liège, Institut de mécanique et Génie Civil, Bât. 52 1 Chemin des chevreuils, B-4000 Liège 1, Belgium

ARTICLE INFO

Article history:
Received 26 September 2013
Received in revised form
3 June 2014
Accepted 5 June 2014

Keywords:
Second gradient
Concrete
Damage
Strain localization

ABSTRACT

Being a quasi-brittle material, concrete under tensile loading exhibits a strain softening behaviour that cannot be accurately reproduced with classical (without an internal length parameter) continuum mechanics models. An internal length parameter must be introduced to regularize the problem, as in the case of the so-called second gradient model. In this approach, an enriched kinematic description of the continuum is adopted considering higher (second) order gradients of the displacements following the work of Cosserat, Toupin, Mindlin and Germain. The model has been developed by Chambon and co-workers and has been mainly used with plasticity constitutive laws to reproduce the non-linear behaviour of soils. It is here applied for the first time to concrete and reinforced concrete specimens considering material laws based on the damage mechanics theory. The advantages and limitations of the approach are discussed, and possible improvements towards more realistic responses are suggested.

1. Introduction

Since the 70s [1–3] researchers study the strain localization in quasi-brittle materials, or more generally in materials exhibiting strain softening. Strain localization zones are clearly observed in experimental tests [4] and it is well known that they cannot be modelled with classical (without an internal length parameter) continuum mechanics models. Analytically, the differential operator becomes hyperbolic and an infinite number of solutions are possible. Numerically, the loss of ellipticity appears as a pathological mesh dependency of the results. These shortcomings are due to the lack of an internal length parameter in the continuum model that characterizes the width of the localization zone [5–11]. Different approaches exist in the literature to regularize the problem and to obtain objective numerical global (i.e. forces, displacements) and local (i.e. strains, stresses, internal variables) results. The first is important for design purposes and the second to deal for example with durability and crack propagation problems. The different approaches are briefly summarized hereafter (see also [12] for a more detailed literature review):

- *Regularization based on energy*: The principle is to keep the same fracture energy dissipated during the formation of cracks whatever the size of the finite element mesh [13–16]. For this, the post-peak behaviour of the adopted constitutive law is changed according to the size of the finite elements. This approach provides global results that may seem to be independent of the size of the mesh. Nevertheless, the localization zone is necessarily concentrated in one element (as in a classical continuum mechanics model without an internal length parameter) and thus local and global results are not objective. Results are also dependent on the orientation of the finite element mesh.
- *Regularization based on time dependency*: Viscous terms are introduced in the model that restore the ellipticity of the differential operator [17]. However, because this method does not introduce an internal length to control the width of the localization zone, severe mesh dependence is avoided for dynamic but not for quasi-static calculations [18].
- *Regularization based on spatial dependency*:
    - Using a non-local integral type variable (i.e. on the damage parameter or on the equivalent strain for constitutive laws based on damage mechanics) [8]. For this integral type model, the interaction between material points across a crack [19] can still pose problems. Similar difficulties also exist for materials presenting a different behaviour in traction and compression (consider for example the interactions across the compression and traction zones for a concrete beam

* Corresponding author.
E-mail addresses: Gwendal.Jouan@ec-nantes.fr (G. Jouan), Panagiotis.Kotronis@ec-nantes.fr (P. Kotronis), f.collin@ulg.ac.be (F. Collin).

http://dx.doi.org/10.1016/j.finel.2014.06.002
0168-874X/

submitted to bending, see also Section 4). Different approaches can be found in the literature to deal with these problems, mainly consisting in modifying the adopted weight function either near the boundaries [20] or by introducing a dependence on the stresses [21]. Nevertheless, this last assumption implies that the internal length is no longer a constant material parameter but that it decreases with increasing loading. Furthermore, as is the case for the other regularization techniques, the ability of the method to reproduce accurately global and local results under size effect needs to be more thoroughly studied [22].

- Using strain gradients controlling the evolution of the internal variables (i.e. the second gradient of the plastic strain in the consistency condition and/or the flow rule) [7,23]. This type of model is shown to be equivalent to the integral type model [23].
- Alternatively, the nonlocal variable can be defined via an implicit gradient of the corresponding local variable, and is then the solution of a boundary value problem [24]. This type of model is shown to be equivalent to the integral type model [23].
- By taking into account gradient of internal variables (the damage variable in the case of damage models) in the energy [25–27]. The gradient term here acts as a penalization term for the cases of high localization.
- More recently, strainlocalization due to damage has been treated using the thick level set approach [28]. The level set separates the undamaged from the damaged zone while the damage variable and its growth are a function of the level set propagation. The force driving the damage front is nonlocal in the sense that it averages information over the thickness in the wake of the front [28].
- A rather natural way of introducing (indirectly) a length parameter in a continuum model is to account for the microstructure of the material. The general class of the so-called microstructured models or higher order continuum models allows for the description of the kinematics of the microstructure by using an additional tensor in the displacement field. Higher order continuum theories can be traced back to the works of the Cosserat et al. [29], Toupin [30] and Mindlin [31,32] and have been generalized and properly formulated by Germain [33,34] using the virtual power method.

In this paper, we choose to work with the second gradient model developed by Chambon and co-workers [35–39]. This model can be seen as a particular case of a higher order continuum (see Section 2) and has been mainly used till now to regularize problems involving strain localization in soils. It is used hereafter to concrete and reinforced concrete elements. The paper is structured as follows: the theoretical framework of the second gradient model and its numerical implementation are at first presented. The objectivity of the numerical results is shown for a 1d concrete specimen and the evolution of the localization zone is discussed. The paper ends with a case study, the simulation of a three point bending test on a reinforced concrete beam. Discussion on the numerical results shows the advantages and limitations of the approach that should be considered as a first step towards the use of local second gradient models for concrete structures.

2. The second gradient model

2.1. Theoretical framework

As detailed in the seminal work of Germain [33,34], using the virtual power method one can choose a field of virtual displacements to describe the proper kinematics of a higher order continuum including its microstructure. The internal stresses, limit conditions and equilibrium equations appear naturally as long as the linear form representing the virtual power is correctly defined and that it respects the principle of material independence.

The second gradient model developed by Chambon et al. [35,36] can be seen as a particular case of a higher order continuum where up to second gradient terms are adopted and the macrostrains are considered to be equal to the microstrains. The authors have come to this assumption following experimental results that showed that for the case of geomaterials microrotations equal macrorotations [40,41]. They have presented case studies in the framework of plasticity and have shown that this type of model restores mesh objectivity but not the uniqueness of the solution [37–39,42].

For the second gradient model, the virtual displacement field must be chosen as a field of continuous and continuously differentiable velocities. According to the general theory for continua with microstructure presented in [37] and assuming that microstrains are equal to macrostrains, the virtual work principle equation takes the following form (for any α, $\alpha^\star$ defining the virtual quantity). For the sake of simplicity, we neglect hereafter the body force terms and the presentation is done for a 2d continuum:

$$\int_\Omega \left(\sigma_{ij} \frac{\partial u_i^\star}{\partial x_j} + \Sigma_{ijk} \frac{\partial^2 u_i^\star}{\partial x_j \partial x_k} \right) d\Omega = \int_\Gamma (p_i u_i^\star + P_i D u_i^\star)\, d\Gamma, \tag{1}$$

with

- i, j and k (varying from 1 to 2),
- x_i the coordinates,
- u_i the macrodisplacements field,
- Dq the normal derivative of any quantity q:

$$Dq = \frac{\partial q}{\partial x_k} n_k, \tag{2}$$

- σ_{ij} the Cauchy stresses (macrostresses),
- Σ_{ijk} the double stresses,
- p_i the classical traction forces,
- P_i the double traction forces,
- Γ the boundary of Ω.

The Cauchy stress σ_{ij} is, as in classical continua, symmetric, the double stress Σ_{ijk} is symmetric with respect to its indices j and k. Application of the virtual work principle equation (1) and two integrations by parts provide the balance equation and the boundary conditions. The balance equations become

$$\frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial^2 \Sigma_{ijk}}{\partial x_j \partial x_k} = 0. \tag{3}$$

Assuming that the boundary is regular (which means existence and uniqueness of the normal for every point of the boundary Γ of the studied domain), after one more integration by parts, we get

$$\sigma_{ij} n_j - n_k n_j D\Sigma_{ijk} - \frac{D\Sigma_{ijk}}{Dx_k} n_j - \frac{D\Sigma_{ijk}}{Dx_j} n_k + \frac{Dn_l}{Dx_l} \Sigma_{ijk} n_j n_k - \frac{Dn_j}{Dx_k} \Sigma_{ijk} = p_i, \tag{4}$$

and

$$\Sigma_{ijk} n_j n_k = P_i, \tag{5}$$

where p_i and P_i are prescribed. The tangential derivative of any quantity q is defined by

$$\frac{Dq}{Dx_j} = \frac{\partial q}{\partial x_j} - \frac{\partial q}{\partial x_k} n_k n_j.$$