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A new and simple locking-free triangular thick plate element using independent shear degrees of freedom

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ABSTRACT

In this paper, a new locking-free element triangular thick plate element with 9 standard kinematic degrees of freedom and 6 additional degrees of freedom for shear strains (TTK9S6) for analyzing plate/shell structures of thin or thick members is developed. With an appropriate use of independent shear degrees of freedom (DOF), the shear locking problem is completely removed without inducing any numerical expediency such as reduced integration, assumptions of strains/stresses, nor are additional efforts needed to stabilize spurious zero energy modes. Compared to existing triangular shear-deformable plate elements that pass patch tests for both thick and thin plates, the formulation of the present TTK9S6 element is very simple—and perhaps as simple as possible. A number of numerical examples are tested showing the convergence and accuracy of the TTK9S6 element.

1. Introduction

Plate/shell structures are widely used in civil, naval and aerospace engineering. For the stress analysis of plate/shell members in the design of structures, two main theories, namely Kirchhoff plate theory and Mindlin–Reissner plate theory, have been developed. It is well known that a plate model based on Kirchhoff plate theory, denoted as Kirchhoff plate, is appropriate for the modeling of thin structures where shear deformation can be ignored. Unfortunately, the finite element discretization of Kirchhoff plate theory requires C^1 continuity of the interpolation, which is a significant drawback. Conversely, Mindlin–Reissner plate theory for the analyses of thin and thick structures requires only C^0 continuity for the transverse displacement and the normal rotations, and thus the difficulties of constructing C^1 interpolations are avoided. However, Mindlin–Reissner plate elements with displacement DOF only produce poor results in the thin plate limit because of the shear locking phenomenon. In order to eliminate shear locking from Mindlin–Reissner plate elements, reduced integration, assumed natural stain approach, and assumed stress elements (also known as mixed/hybrid stress elements) have been proposed. For example, we mention the reduced integration method by Zienkiewicz et al. [1] and Pugh et al. [2], the selective integration method by Malkus and Hughes [3] and Hughes et al. [4] the mixed interpolation of tensorial components (MITC) element family by Bathe and Dvorkin [5], the Discrete Shear Triangular (DST) element family by Batoz and Lardeur [6] and Batoz and Katili [7] which was later extended by Zienkiewicz et al. [8], and the mixed interpolation and smoothed curvatures (MISC) element by Nguyen et al. [9], the refined 9-Dof triangular Mindlin plate elements (RDKTM) by Chen and Cheung [10], the hybrid-Trefftz plate elements by Choo et al. [11], the assumed stress/strain elements by Lee and Pian [12], Katili [13], Brasile [14] and the "discrete shear gap" (DSG) technique [15,16] as a more general triangular plate element employing displacement and shear DOFs at each node by assuming the 'natural strain' mode. However, most of these elements introduced new problems of spurious energy-free modes. Furthermore, the formulation or the numerical implementation of such elements is usually complex.

In recent years, meshless approaches have drawn much attention in plate analysis due to their higher order continuity of approximation in a global sense, and they have been used to develop higher order continuous plate element, to eliminate the excessive degrees of freedom and to successfully circumvent shear locking in thick structures. Donning et al. [17] used the cardinal spline function to construct C^2 meshless approximation which is valid for modeling thin and moderately thick plates. Le et al. [18] used the MLS approximation for the construction of smooth pure moment fields to calculate the lower bound limit load of plates using only nodal displacements as unknowns. Kanok-Nukulchai et al.

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presented a method for completely eliminating the presence of transverse shear locking in shear-deformable plates by applying the Element-free Galerkin (EFG) Method [19]. In Ref. [20], a locking-free Meshless Local-Petrov Galerkin (MLPG) formulation is presented for shear-deformable thick plates. Moreover, the applicability of a meshless approach in problems of crack propagation and large deformation analyses appears to be far more flexible than that of the standard FEM such as the thin shell formulation for modeling dynamic fracture [21]. However, the field reproduced by a meshless approximation is usually non-polynomial in a global sense and theories developed so far in the context of FE for error control in many cases do not directly apply to meshless approximations [22] unless a series of analyses is carried out a posteriori—however, this has significant adverse implications on computing time. Moreover, the computational cost in obtaining shape functions and performing numerical integration with acceptable accuracy is overall much more expensive than in the FEM. There are growing interests at present in developing the so-called smoothed FEM for shell analysis where the integral of shear strain only appears on element edges. Other novel method including SFEM [23,24], phantom node method [25] and NURBs based FEM [26] are also exploited by utilizing their advantages such as smoothing approximation, better geometric representation of curved structures or flexibility in modeling arbitrary crack propagation in shells.

Although the use of these novel numerical method can be promising, in this paper we wish to focus on finite element techniques, since the finite element method remains the most widely used method of spatial discretization in solid and structural mechanics. Thus, we present a new locking-free element, named TTK9S6, which is effective for analyses of plate/shell structures comprising of thin or thick members. The present element is a three-noded plate element with 5 DOF per node, consisting of 1 transverse displacement, 2 derivatives of the transverse displacement around two independent axes, and 2 transverse shear strain DOF. The shear locking problem is completely removed in the TTK9S6 element, without any numerical expediency such as reduced integration, the use of assumed strains/stresses, or the need for the stabilization of spurious zero energy modes, by an appropriate use of the independent shear DOF, as discussed in Refs. [27,28]. Notably, in Ref. [28] the formulation is based on the DST element while the independent shear degrees of freedom are used. Here, we do not follow the DST formulation and instead a completely new triangular plate element is developed using 9 kinematic degrees of freedoms and 6 shear degrees of freedom. It will be shown that the present TTK9S6 has, arguably, the simplest formulation compared to all the other existing triangular shear-deformable plate elements that can pass patch tests for thick as well as thin plates. The present element is capable of removing the shear locking problem, it is however necessary to mention that the convergence rate of finite elements does not necessarily depend on the locking-free condition. Though the present formulation of triangular element introduces additional DOFs per node, it has a general formulation from thin to thick plates. There is no equivalent formulation in the literature to the authors knowledge compared to the present formulation.

2. Interpolation and formulation of the TTK9S6 element

In this section the interpolation of the new TTK9S6 element will be described. We will start with the decomposition of the transverse shear strain from the total rotation.

Consider a typical three-noded triangular plate element of which the side view is shown in Fig. 1. The derivatives of the transverse displacement w with respect to two independent axes,

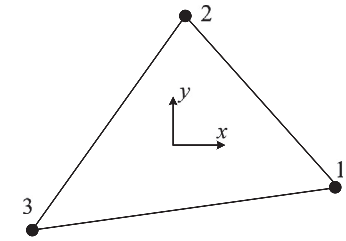

Fig. 1. Three-noded triangular plate element.

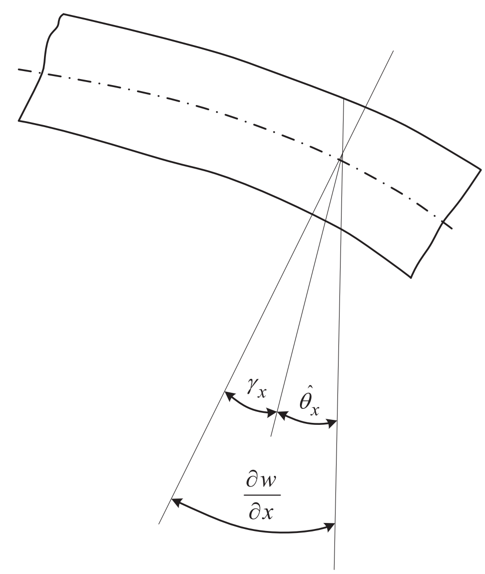

Fig. 2. Decomposing derivatives of transverse displacement into transverse shear strain and rigid body rotation.

without separating the influence of transverse shear deformation, are expressed as

$$\theta_x = \frac{\partial w}{\partial x}, \theta_y = \frac{\partial w}{\partial y} \tag{1}$$

here θ_x will include the contribution from the rigid body rotation of the plate around the y axis denoted as $\hat{\theta}_x$. Similarly, θ_y contains a part $\hat{\theta}_y$ representing the rigid body rotation of the plate around the x axis. In this way, the influence of the transverse shear deformation can be separated from rotation as shown in Fig. 2. Thus the transverse shear strains of the element can be expressed by

$$\gamma_x = \theta_x - \hat{\theta}_x, \gamma_y = \theta_y - \hat{\theta}_y \tag{2}$$

In the following, the shape functions used for the transverse displacement, rotation and shear strain in the TTK9S6 element will be described. In the derivation, additional mid-side nodes are employed as shown in Fig. 3. These mid-side nodes are only used for calculating shape functions and do not lead to additional global unknowns as it was the case in the study on DST elements [7,8]. Now we define the following shape functions:

$$\phi_i = L_i,\ \phi_{xi} = \frac{1}{2}(x - x_i)L_i,\ \phi_{yi} = \frac{1}{2}(y - y_i)L_i \tag{3}$$

$$R_i = \left(\sum_{k=4}^{6} N_k\right)\phi_{i,x},\ R_{xi} = N_i + \sum_{k=4}^{6}(N_k\phi^k_{xi,x}),\ R_{yi} = \sum_{k=4}^{6}(N_k\phi^k_{yi,x}) \tag{4}$$

$$Q_i = \left(\sum_{k=4}^{6} N_k\right)\phi_{i,y},\ Q_{xi} = \sum_{k=4}^{6}(N_k\phi^k_{xi,y}),\ Q_{yi} = N_i + \sum_{k=4}^{6}(N_k\phi^k_{yi,y}) \tag{5}$$