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Formulation and application of the adaptive hydraulics three-dimensional shallow water and transport models

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ABSTRACT

Next generation, conservative finite element hydrodynamic and transport models are vital for accurate and efficient ocean, estuary and river simulation. Numerical models such as these have been developed for decades by the U.S. Army Corps of Engineers at the Engineering Research and Development Center (ERDC). This paper focuses on recently developed implicit, multi-dimensional finite element 3D shallow water and transport models included in the Adaptive Hydraulics (AdH) numerical suite. These AdH 3D models benefit from their adaptive meshing capabilities to resolve sharp solution gradients, such as those often encountered with baroclinic wedges traveling up an estuary channel. This paper presents the AdH 3D mathematical formulation and solution procedure used to solve the weak finite element equations for these models, along with results for several common verification cases and an AdH Galveston Bay validation study. A novel Streamline Upwind Petrov–Galerkin (SUPG) method for 3D shallow water models is described which reduces to the AdH 2D shallow water SUPG formulation under certain conditions. Careful attention is placed on ensuring discrete consistency in the equation set.

1. Introduction

The shallow water (SW) equations are a set of hyperbolic partial differential equations that describe hydraulic flows where the horizontal length scale is much greater than the vertical length scale, and the Bousinnesq approximation is valid [1]. Under these conditions, conservation of mass implies that the vertical inertia of the fluid is small, and the vertical pressure gradients are nearly hydrostatic. Thus horizontal pressure gradients are due to the displacement of the pressure surface only. Although counterintuitive to their name, shallow water equations are frequently used to predict both ocean and estuary dynamics, since these bodies of water are often large enough to propagate long wavelengths with respect to their depths (see [2–4], for some examples).

Models for predicting the SW equations are often accompanied by some number of additional scalar transport models whose constituent solutions depend on the water flux. If the transported constituents do not feedback into the hydrody-

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namics, the application is *barotropic*. In a *baroclinic* application, on the other hand, transported constituents, such as salt and temperature, may have an impact on the water density from pressure gradients that subsequently drive flow in a two-way coupled feedback between the transport and SW equations. High concentrations of sediment may also have the same affect. Coupling between the hydrodynamics and constituent transport is usually done in a split-operator sense, where density influences are incorporated into the hydrodynamics at the next time step [5–8].

In general, analytic solutions to the SW and transport equations are not possible, however, discrete, numerical methods, such as finite element (FE) or finite volume (FV) methods, can be used to provide high fidelity computational solutions to complex applications. Finite element/volume methods are capable of handling unstructured meshes and offer flexibility in the order of interpolation (they can be low/high order, discontinuous/continuous, etc.). Such advantages are extremely useful for shallow water models, where, for example, irregular coastlines and bathymetries must be captured. This study focuses on the Adaptive Hydraulic (AdH) finite element suite for solving shallow water and transport problems. AdH is a Department of Defense high fidelity, finite element resource for 2D and 3D dynamics. It supports a host of features vital to most hydraulic and transport-engineering applications, including baroclinic capabilities, surface wave and wind-wave stress coupling, flow through hydraulic structures (weirs, flap gate, etc.) and vessel flow interactions. To remain robust in the general field of environmental quality modeling, AdH has been internally linked to process-oriented libraries for (1) cohesive/non-cohesive sediment transport [9–12], (2) water/nutrient contaminant transport [13], (3) meteorological data [14], (4) friction modeling [15–18] and (5) turbulence modeling [19]. The AdH suite has a wide user base, including USACE water districts, universities and consulting agencies (see [20–23,12] for some example applications). The suite engine is based on an implicit, stabilized finite element method. All models within the AdH suite are both spatially and temporally adaptive.

In shallow water applications with complex lateral velocity depth profiles, 3D solutions are required. A minimal number of 3D SW models exist today which are robustly applicable, unconditionally stable with respect to time-stepping and easy to implement. Some of the more popular production models include: the semi-implicit models Adcirc-3D [7], [6] and SELFE [5] and the implicit model SLIM [8]. Herein we describe a new addition to the AdH model suite, a stabilized and continuous Petrov–Galerkin finite-element formulation used for solving 3D SW and transport models. The 3D models presented here are the production models for the Army Corps of Engineers, and the models have already been successfully applied to a number of complex estuaries. It should be noted that the mathematical foundations presented herein are very similar to that of SLIM [8]. Both solve the 3D shallow water equations implicitly in primitive form (though SLIM offers a semi-implicit option using mode-splitting). Some notable differences, however, are as follows: (1) AdH allows for both tetrahedral and triangular prism elements, (2) AdH is not restricted to the same number of vertical layers because of this element flexibility, (3) AdH supports spatial adaption and (3) SLIM is stabilized using mixed formulations based on $P_1^{NC} - P_1$ or RT_0 pairs [24–26], while AdH utilizes linear solution expansions and Streamline Upwind Petrov Galerkin (SUPG) stabilization [27,28].

Pivotal to the success of a SW numerical model is stability in advection-dominated regions. Due to the hyperbolic form of the SW and transport equations, symmetric FE methods can be subject to spurious oscillations around sharp solution gradients [29]. In the worse case, these numerical oscillations can propagate throughout the solution domain and cause model instability. These node-to-node wiggles (sometimes called $2\triangle x$ wiggles) have plagued advection–diffusion modelers for years, and a vast amount of peer-reviewed literature throughout the last three decades has been dedicated to their source identification and mitigation [30–34,2,28]. As mentioned, the numerical oscillations are most notable in regions with strong boundary layer effects or highly advective flows and around solution shocks. In such cases, as the advection terms grow, there is a reduction in the number of boundary conditions which causes a rapid change in the solution (i.e. a sharp boundary layer) near the region of the lost boundary condition. It is now well known that these issues arise from the implementation of symmetric FE weight/test functions [27], and some numerical methods have been established that help suppress these oscillations for the shallow water equations specifically. The most time-tested of these include the following: implementation of a generalized wave continuity equation (GWCE) [32,33], streamline upwind Petrov–Galerkin (SUPG) methods [27,28], mixed basis formulations [24–26] and discontinuous Galerkin (DG) methods [34,2]. To avoid basis compatibility issues between the unknowns in the SW and transport equations, AdH uses discrete linear expansions for all independent variables, restricting stabilization options to using the GWCE, DG methods or SUPG methods. Each of these stabilization methods, however, comes at a cost to model efficiency, applicability and accuracy.

The first developments of FE shallow water models were based on the wave continuity equation [30], wherein the primitive shallow water equations are manipulated to form a wave equation for the free-surface elevation. The 2nd-order and symmetric wave equation was utilized to suppress primitive spurious oscillations without resorting to artificial damping. In time, however, it was found that the wave continuity equation was subject to significant mass balance errors in applications where the SW nonlinear terms were significant [31]. To mitigate these errors, the generalized wave continuity equation (GWCE) was formulated and is still used by many modern shallow water solvers today [32,33]. The GWCE corrects a previous inconsistency in the wave equation advection terms and allows a user-tunable "mixing" of the wave equation with the primitive continuity equation. Despite their success, however, GWCE models are still subject to advection instabilities [33], particularly as the primitive continuity equation contribution is increased, and, because the primitive form is not satisfied directly in the discrete sense, mass imbalances may still arise [35,36]. These GWCE issues can significantly reduce the accuracy of an application, particularly when solving transport problems and long-time simulations where a tight mass balance is essential.

An alternative to GWCE stabilization is to solve the primitive equations with a discontinuous Galerkin (DG) FE method. Discontinuous Galerkin models for general hyperbolic, and more specifically shallow water models, are relatively new, but