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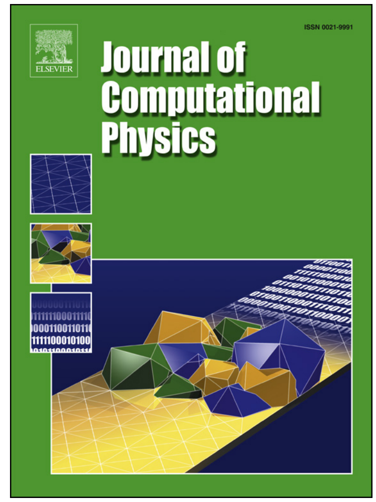

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A C^0IP method of Transmission Eigenvalues for Elastic Waves

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Abstract

This paper develops a discontinuous Galerkin method to compute a few smallest elasticity transmission eigenvalues, which are of practical importance in inverse elastic scattering theory. For high-order problems, comparing with classical conforming finite element methods, discontinuous Galerkin methods use simpler basis functions which make the numerical implementation much easier. In this paper, we propose an interior penalty discontinuous Galerkin method using C^0 Lagrange elements (C^0IP) for the elastic transmission eigenvalue problem and prove the optimal convergence. Numerical examples are presented to validate its effectiveness. Both real and complex eigenvalues can be obtained.

Keywords: Transmission eigenvalue problem, Elastic wave equation, Discontinuous Galerkin method

1. Introduction

Transmission eigenvalues play an increasingly important role in inverse scattering theory. For example, transmission eigenvalues can be used to obtain the physical properties of the scattering targets [7, 29]. They are also used for the uniqueness and reconstruction in the inverse scattering theory. The readers can refer to the *Special Issue of Inverse Problems on Transmission Eigenvalues* (Number 10, 2013) and references therein for the rapid development on this research topic.

This problem is very challenging since it is nonlinear and non-selfadjoint. Similar to the cases of acoustic and electromagnetic waves, the elasticity transmission eigenvalue problem is very important in the qualitative reconstruction methods for inhomogeneous elastic media. There exist only a few studies on the elasticity transmission eigenvalue problem [9, 10, 3, 2]. In [2], under some conditions on elastic tensors and mass densities, the authors show that there exists a countable set of elasticity transmission eigenvalues.

We focus on the numerical methods in this paper. Numerical methods for the acoustic and electromagnetic transmission eigenvalues have been developed by many researchers recently [11, 28, 19, 1, 23, 20, 8, 24, 32, 26, 30, 15]. It is highly non-trivial to develop finite element methods for the transmission eigenvalue problems in general since the problem is nonlinear and non-selfadjoint [31]. The general finite element discretization usually leads to non-Hermitian matrix eigenvalue problems. It is challenging to compute (interior) generalized eigenvalues for non-Hermitian matrices. In particular, when the size of matrices is large and there is no a priori information of the spectrum, classical methods in numerical linear algebra would fail. New methods have emerged to treat such difficult problems [16, 17].

The goal of this paper is to develop effective numerical methods to compute a few smallest transmission eigenvalues for elastic waves, which can be used to estimate material property of the elastic body (see, e.g., [29]). To our knowledge, there is only one work on the numerical treatment of transmission eigenvalues for elastic waves. In [18], the authors construct a nonlinear function whose values are generalized eigenvalues of a series of self-adjoint fourth order problems. The roots of the function are the transmission eigenvalues. However, only real eigenvalues can be captured. In this paper, we employ the C^0IP method for the transmission

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