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"On the fly" stabilization of the Coarse-Mesh Finite Difference acceleration for multidimensional discrete-ordinates transport calculations

Emiliano Masiello

Commissariat à l'Énergie Atomique et aux Énergies Alternatives (CEA-Saclay) DEN/DANS/DM2S/SERMA, CEA de Saclay, 91191 Gif-sur-Yvette Cédex, France

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ABSTRACT

In this paper, the Jacobian matrix of the Coarse-Mesh Finite Difference (CMFD) method is analyzed. Both the homogenization and the current preservation effects are studied in heterogeneous multidimensional configurations. Some bounding values of the spectral radius are also given. An analytical stability analysis is carried on an interface slab problem. This analysis leads to the computation of the stability parameter introduced in the Generalized Coarse Mesh Rebalancing method. A dynamical stabilization technique is proposed for multidimensional neutron lattice calculations. Numerical calculations show that the proposed technique dumps the unstable modes, in particular in optically thick configurations, where the classical CMFD method fails to converge.

1. Introduction

The Coarse-Mesh Finite Difference (CMFD) method is an efficient and effective non-linear technique to accelerate discrete ordinates source iterations (SI). The iterative scheme is a two-level multigrid: the discretized transport operator is the fine operator that feeds the coarse diffusion operator at each iteration [1,2]. The equivalence transport-diffusion is established by dynamically preserving neutron balance on the coarse regions. This entails a flux-weighted homogenization for the cross sections and an artificial flux-current equation at interfaces of the CMFD meshes. The flux-current equation has a typical finite-difference form, which is adjusted with a transport-computed parameter guaranteeing the conservation of net interface currents. A specific finite-difference flux-current closure characterizes the version of the CMFD method [3]. This equation has an impact on the quality of the diffusion matrix, which is, in general, no longer symmetric. The coarse operator results in a sparse matrix of reduced dimensions in space, in angle and in energy, for multigroup applications. The CMFD has been successfully implemented in diffusion codes to accelerate multigroup iterations [2,4]. Years after, it was also applied to transport codes to speed up both SI and multigroup power iterations, with the possibility to perform also quick sensitivity studies [5,6].

However, the convergence of the CMFD is not assured. The acceleration can fail for very thick or very thin homogeneous regions in scattering-dominated regimes, as confirmed in the literature [6–8]. The Fourier analysis on infinite homogeneous problems, obtained by linearizing the scheme in the neighborhood of the fixed point, demonstrates that the acceleration is effective and stable only when the optical thickness of the coarse mesh is less than one mean free path (mfp). This recommendation for ensuring stability was extended also to heterogeneous calculations. However, in practical multigroup

E-mail address: emiliano.masiello@cea.fr.

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lattice problems, it is not always convenient or possible to reduce the size of the coarse mesh to match the stability condition.

Based on the definition of two non-linear parameters, Yamamoto proposed an artificial flux-current equation to overcome non-converged cases [9]. The first parameter preserves the transport currents while the second modifies the finite-difference diffusion coefficient to prevent instabilities. This method was defined as the Generalized Coarse Mesh Rebalancing (GCMR) since, depending on the value of the second parameter, it can behave as the CMFD or as the Coarse Mesh Rebalancing (CMR). Yamamoto verified the effectiveness of the stability parameter by a Fourier analysis on an infinite homogeneous slab for different values of the optical thickness. The spectral radius of the iterative scheme has been evaluated as a function of the stability parameter allowing for the choice of the optimal parameter for a given problem configuration. This research has brought to light the possibility for stabilizing the scheme.

Jarrett and his coauthors suggest that several free transport source iterations prior to the application of the CMFD can stabilize the non-linear scheme. In this case, the transport feeds the non-linear operator with a 'better' solution, but the computational time may increase substantially because of the increasing number of transport sweeps [10]. Two techniques based on the modification of the diffusion coefficient are proposed: the first is the under-relaxation of the current correction factor, while the second, namely Artificial Grid Diffusion, consists in modifying the diffusion coefficient with an additional term which is the product of the spatial thickness of the mesh times a user's parameter.

Lulu Li and her coauthors proposed a linear correction factor that takes into account the dependency of the average-flux from the neighboring coarse-cells [11]. Although the proposed technique improves the convergence speed of the method, it is deeply linked with the discretization of the fuel pin cell in sectors and it can not be generalized to any type of geometrical discretization.

In this paper, an explicit calculation of the Jacobian matrix is proposed to investigate instability issues. The research gives a clear view on the source of instabilities and allows for a viable strategy for a dynamical calculation of the stability parameter. More precisely, the iterative scheme is viewed as a multivariable explicit dynamical system of the type $\phi^{(i+1)} = \mathcal{F}(\phi^{(i)})$, where i is the iteration index and $\phi^{(i)}$ and $\phi^{(i+1)}$ contain the region-wise average scalar flux at two successive iterations. $\mathcal{F}(\phi)$ summarizes the transport-diffusion nonlinear map, which is, in our case, infinitely differentiable except for regions in which $\phi = 0$. The analysis of the Jacobian allows for the investigation of the stability close to and far from the fixed point [12]. The analysis is carried on separately for the interface-current conservation and for the cross-section homogenization. In particular, two limit cases for the CMFD mesh are analyzed. The first is with the coarse mesh identical to the fine mesh: this eliminates the homogenization effects and keeps the nonlinearity introduced by the interface-current conservation. The second is with the fine mesh homogenized in a single coarse region: this keeps the non-linearity introduced by the flux-weighted cross-sections. The analysis is performed by using a simple parametrization of cross sections allowing for the investigation of various heterogeneous configurations. Next, a stability parameter is introduced in a modified finite-difference diffusion coefficients following the GCMR technique. An "on the fly" technique for computing the stability parameter is proposed. The method, which is currently applied in multigroup lattice calculations, is the result of an analytical asymptotic study of an interface slab problem [13]. Numerical tests confirms improvements of the CMFD performances in both stability and effectiveness.

The remainder of this paper is divided as follows. Sections 2 introduces the notation for the CMFD matrix and the Average Flux Correction (AFC) scheme. Section 3 is dedicated to the calculation of the Jacobian matrix. Sections 4 and 5 analyze respectively the interface-current effect and the homogenization effect. In Section 6, the stability parameter is introduced and computed on a 2-node interface problem. In Section 7, the effectiveness of the stability parameter is evaluated on a heterogeneous 2D checkerboard problem studied with uniform and non-uniform flux distribution. In Section 8, a simplified strategy for multidimensional lattice problem is exposed. Finally, Section 8 and 9 contain respectively the verification of the stabilization strategy on the EIR2 benchmark and the conclusions. For completeness, several results have also been included in the Appendices: Appendix A describes the transport matrices used for the solution of the 2-node slab problem, Appendix B contains the calculation of approximated bounds for the spectral radius, Appendix C contains the study of the homogenization effect on a 2-node slab problem and, finally, Appendix D contains the analysis of infinite homogeneous configurations usually studied in the Fourier analysis.

2. CMFD operator with AFC drift closure

The CMFD operator is derived on a coarse mesh where each node is composed of one or more contiguous transport mesh regions. The CMFD quantities, as the integrated fluxes and currents, will be denoted by the subscript d, standing for diffusion. For convenience, we will use a region-wise numbering for the interfaces. The starting point of the CMFD operator is the node balance:

$$\sum_{k'\cap k} J_{d,k\to k'} + \Sigma_{d,a,k}\phi_{d,k} = Q_{d,k}, \tag{1}$$

where k is the coarse-node index, the sum in k' is over all the nodes adjacent to node k indicated as $k'\cap k$, $J_{d,k\to k'}$ is the surface-integrated net neutron current on the outgoing normal from k to k', and $\phi_{d,k}$ and $Q_{d,k}$ are respectively the volume-integrated scalar flux and the external source in node k. Also, in Eq. (1), $\Sigma_{d,a,k}$ is the absorption cross section in node k, which is obtained by flux-weighting homogenization from the transport calculation: