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A boundary integral method for modelling vibroacoustic energy distributions in uncertain built up structures

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ABSTRACT

A phase-space boundary integral method is developed for modelling stochastic high-frequency acoustic and vibrational energy transport in both single and multi-domain problems. The numerical implementation is carried out using the collocation method in both the position and momentum phase-space variables. One of the major developments of this work is the systematic convergence study, which demonstrates that the proposed numerical schemes exhibit convergence rates that could be expected from theoretical estimates under the right conditions. For the discretisation with respect to the momentum variable, we employ spectrally convergent basis approximations using both Legendre polynomials and Gaussian radial basis functions. The former have the advantage of being simpler to apply in general without the need for preconditioning techniques. The Gaussian basis is introduced with the aim of achieving more efficient computations in the weak noise case with near-deterministic dynamics. Numerical results for a series of coupled domain problems are presented, and demonstrate the potential for future applications to larger scale problems from industry.

1. Introduction

Noise and vibration simulations for mechanical structures are commonly performed using numerical solvers for linear wave equations [1,2]. The underlying numerical technologies are typically based on finite element methods, finite volume methods, boundary element methods or a variety of spectral methods. There are, however, two fundamental limitations when numerically approximating the solutions of wave equations directly in this manner. Firstly, the size of the numerical models required to obtain reliable results will eventually grow large enough to become computationally prohibitive when the local wavelengths become significantly smaller than the dimensions of the physical system. Secondly, the modal density increases with the frequency and, as a consequence, the vibrational responses of "identical" manufactured structures from the same production line can differ greatly in the high frequency regime. That is, uncertainties play a more important role when the structural modes become sufficiently dense that they can exchange positions due to small structural differences within standard manufacturing tolerances. For these reasons statistical methods for predicting averaged energy distributions, such as Statistical Energy Analysis (SEA) [3], have become popular tools for high frequency noise and vibration simulations [4]. However, the underlying assumptions of SEA are often hard to verify *a-priori*, and the method only provides a coarse description of the structure under consideration since constant energy levels are assumed throughout relatively large substructures of the overall model [5].

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An alternative framework that can provide a more detailed description of the vibrational or acoustic energy distribution is to model wave energy transport at high frequencies via a geometrical ray description, which neglects interference and other wave effects. The underlying wave problem is then reduced to tracking densities of rays or particles in phase-space and becomes part of a wider class of mass, particle or energy transport problems driven by an underlying deterministic velocity field. The computational cost of direct ray or beam tracing methods, based on following rays or beams from a source to a receiver, scales with the number of paths that must be modelled. For complex domains including many reflections, the number of paths can grow so rapidly that applications in room acoustics are typically limited to including at most second order reflections [6]. Furthermore, in complex structures there may additionally be mode conversion and refraction effects to take into account. In these circumstances, indirect methods based on conservation laws such as the Liouville equation can provide a more practical alternative by propagating ray densities (instead of the rays themselves) through phase-space [7–10]. The deterministic propagation of ray densities according to the Liouville equation can be expressed using a transfer operator, known as the Frobenius–Perron operator [11], which transports ray densities along the trajectories of a dynamical system in general (in our case a Hamiltonian ray flow). These ideas have also found their way into the literature in computer graphics [12] and room acoustics [13] amongst others, where the corresponding transfer operator equation is often labelled the rendering equation.

A variety of techniques have been proposed for the discretisation of the Frobenius–Perron operator, with the aim of developing efficient numerical tools for practical applications. Domain based transfer operator approaches involve subdividing the phase-space and approximating the transition rates between these subdivided regions. One of the simplest approaches of this type is known as the Ulam method [14]. For a discussion of convergence properties of the Ulam method in one and several dimensions, see Refs. [15] and [16], respectively. One shortcoming of the Ulam method is that it typically only exhibits sub-linear convergence rates. As a result of this slow convergence and the high-dimensionality of the phase-space, these methods have typically found only limited applications. In order to reduce memory costs and/or speed up the convergence, both wavelet and spectral methods have been proposed [17,18]. A boundary integral reformulation of the Frobenius–Perron operator for a ray flow is derived in Ref. [10], which can be used to determine the stationary ray density (in the long-time limit) corresponding to the high-frequency asymptotic solution of a frequency-domain wave problem. This has the advantage of reducing the dimensionality from a full phase-space model to the Birkhoff coordinates for the phase-space on the boundary.

In this work we consider the ray dynamical modelling of wave energy transport through uncertain structures, which leads instead to a stochastic velocity field driving the energy transport. We propagate ray densities using the corresponding conservation law, here provided by the Fokker–Planck equation for the stochastic evolution of ray densities in phase-space under the action of a noisy flow [11,19]. Direct treatment of the Fokker–Planck equation is often considered infeasible [20], and in this work we will apply a boundary integral formulation of the Fokker–Planck equation for a Hamiltonian flow, where the associated boundary integral operator will take the form of a stochastic evolution operator. Through this approach we achieve a reduction in dimensionality to the boundary phase-space, which makes the corresponding numerical models both smaller and simpler to implement. Stochastic evolution operators have been extensively studied over the last twenty years via periodic orbit techniques [21–25]. Initial work focused on determining spectral properties of the Fokker–Planck operator for Langevin flows in the weak noise limit. However, more recent work has considered higher dimensional cases [26] and the estimation of stationary distributions [27]. Modified Ulam-type methods have also been devised for stochastic transfer operators, see for example Refs. [28,29].

The approach taken here will be based on a modification of a recently-proposed boundary integral reformulation of a stochastic evolution operator, for the case when the noisy flow has been replaced by a noisy boundary map [30]. The resulting boundary integral operator is described in Sect. 2. The modified formulation proposed here has the advantage that it can be generalised from single to multi-domain problems by restricting the range of the noisy boundary map to the edge where the corresponding deterministic map arrives; see Sect. 2.3 for further details. In general, the spatial domain is relatively complex (including corners) compared to the momentum domain, which simply corresponds to the range of angles pointing into the spatial domain at any given boundary point. Note also that highly peaked solutions with respect to the momentum variable are commonplace in ray tracing problems, whereas the solutions typically exhibit a milder dependence on the spatial variable. A local and low order approximation scheme in the spatial variable using piecewise constant collocation is therefore appropriate and furthermore, leads to simplifications in the implementation of the boundary integrals as detailed in Sect. 3.1. For the approximation with respect to the momentum variable we consider two different possibilities for a spectral collocation method in Sect. 3.2; a well-conditioned basis approximation using Legendre polynomials and a radial basis approximation using Gaussian functions. Despite requiring additional preconditioning strategies in order to obtain convergence, the latter has the advantage of providing an exact representation of the typical initial conditions in our proposed model and has the potential for computational cost savings in the case of near-deterministic propagation. These phase-space collocation schemes are detailed throughout Sect. 3 and have the advantage that we can demonstrate the consistency of our numerical implementation with theoretical convergence results for second-kind integral equations with bounded operators [31], as discussed in Sect. 4. We note that these results do not carry over to the Nyström method based discretisation applied in Ref. [30]. Finally, we detail a series of numerical experiments for multi-domain problems in Sect. 5, demonstrating the potential of the proposed methods to model built up structures from industrial applications in high-frequency structural vibrations and acoustics.