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Computational modeling of finite deformation piezoelectric material behavior coupling transient electrical and mechanical fields

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ABSTRACT

This paper develops a finite element formulation and computational model for coupling electric and finite strain dynamic fields with widely discrepant frequencies, governing the behavior of piezoelectric materials. The piezoelectric materials are defined by time-dependent nonlinear constitutive laws. A fully coupled, total Lagrangian finite element formulation is developed for modeling the electric and mechanical fields. A challenge in computational modeling of piezoelectric devices arise due to large discrepancy in the frequencies of the electrical signal and mechanical vibrations, especially when a large number of mechanical cycles need to be simulated. A wavelet transformation induced multi-time scaling (WATMUS) algorithm is developed for dynamic piezoelectric simulations. The WATMUS algorithm projects the high frequency electric fields on to the lower frequency of displacement and velocity fields, on which time integration is performed. The method significantly enhances the computational efficiency in comparison with single time scale integration methods. The accuracy and efficiency of the WATMUS algorithm are validated through piezo-electric applications.

1. Introduction

Piezoelectric materials are increasingly in use in various technological applications, e.g. in actuators, sensors, robotics, energy harvesters, self-healing devices, stretchable electronics etc. [1–3]. Structures of these materials, when subjected to dynamic mechanical and/or electrical loading, develop mechanical and electrical fields that are governed by complex interaction of multi-physics equations. Robust computational models of coupled electro-dynamics phenomena that depict the temporal evolution of electrical and mechanical fields are needed for simulations leading to the design of piezoelectric systems.

Studies have been conducted for understanding the role of electrostatic forces on material response in [4], for active polymers in [5] and dielectric materials in [6–8]. Higher order effects have been observed in piezoelectric and dielectric material properties with large strain fields due to evolving electro-mechanical structure [9]. Constitutive models for piezoelectric behavior in finite deformation have been derived in [10,11]. Classical plate theories have been applied for the

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design of piezoelectric laminates with bending and torsion modes in [12]. Acoustic wave equations for a piezoelectric crystal have been solved using the finite-difference method in time-domain in [13]. Finite element (FE) studies have incorporated piezoelectric-hyper-elasticity constitutive models for finite deformation kinematics in [14]. FE simulations of piezoelectric problems using beam and shell elements and geometrically nonlinear composites have been conducted in [15–17] and [18] respectively. A-posteriori error of FE solutions of thin multi-layer piezoelectric plates has been analyzed in [19]. A beam-like structure with bonded piezoelectric sheets is studied for load transfer in [20], while the actuator model in [21] incorporates both axial and transverse shear forces. A generalized framework for coupling transient electromagnetic and finite deformation dynamic fields has been developed in [22] for predicting the evolution of electrical and magnetic fields and their fluxes in vibrating substrates.

A number of piezoelectric devices, e.g. vibrating energy harvesters [23], smart structures [24], surgical equipments [25], involve transient phenomena with oscillatory electrical signals in a vibrating substrate. Time integration of the cyclic transient response can be a major challenge in modeling coupled multi-physics problems, when disparate time-scales (or frequencies) govern the different physics of the problem. In conventional semi-discrete FE models using single time-scale time integration methods, time-increments are dictated by the higher frequency field. Each cycle of the higher frequency field is discretized into a number of temporal points to meet stability requirements. For transient electromagnetic and dynamic mechanical problems, the frequency ratio of electromagnetic to mechanical loadings can often be very high, e.g. $\frac{\omega_{em}}{\omega_{me}} \sim 10^4$. For these frequency ratios, the single time-scale integration in a coupled analysis requires simulating an inordinate number of electromagnetic cycles to transcend a small number of mechanical cycles. This could be computationally prohibitive. To overcome this challenge, a wavelet transformation based multi-time scaling (WATMUS) method, originally developed for fatigue problems in [26,27], has been extended in [28,29] for coupled FE analysis of vibrating substrates carrying high frequency electromagnetic signals. The WATMUS method projects the high frequency electromagnetic response onto a lower frequency problem through the use of wavelet basis functions. No assumption of periodic response is made in WATMUS. Significant time-scale acceleration has been achieved with this method [26–29].

This paper develops a solution framework for coupled transient electric and dynamic field analysis of nonlinear piezoelectric materials undergoing finite strain vibration and subject to oscillatory electric signals. The governing and constitutive equations for finite deformation piezo-electricity are discussed in section 2. A total Lagrangian finite element formulation is developed in 3. The wavelet transformation-based multi-time scaling (WATMUS) method for 3D piezoelectric problems is formulated in section 4. Numerical examples for two piezoelectric applications are solved in section 5 and the paper concludes with a summary in section 6.

2. Coupled governing and constitutive equations for piezoelectric materials

Coupled electrical and mechanical analysis of piezoelectric materials requires concurrent solution of the governing equations of motion and electric balance laws with nonlinear piezoelectric constitutive relations. In this paper, the finite strain governing and constitutive equations under finite strain kinematics are developed in a Lagrangian description, following electromagnetic-mechanical formulations in [22,28].

2.1. Mechanical and electrical governing equations in Lagrangian description

In this paper, scalar variables are denoted by Roman (a) or Greek letters (α), vectors by boldface Roman letters ($\mathbf{b}$), second-order tensors by boldface italicized Roman letters ($\boldsymbol{S}$) or boldface Greek letters ($\boldsymbol{\alpha}$), third order tensors by special font letters ($\mathbb{P}$) and fourth-order tensors by letters ($\mathfrak{C}$). Subscripts with lowercase $_{(i,j,\cdots)}$ and uppercase variables $_{(I,J,\cdots)}$ correspond to variables in the current and reference configurations respectively. Wavelet coefficients are indexed with lowercase Roman letter superscripts $^{(i,j,\cdots)}$, while finite element node numbers are denoted by Greek letter superscripts $^{(\alpha,\beta,\cdots)}$.

In the finite strain Lagrangian formulation, ${}^0\Omega(=\Omega(0))$ and ${}^t\Omega(=\Omega(t)) \in \mathbb{R}$ correspond to the initial (reference) and current configurations respectively. The corresponding material and spatial coordinates are designated as X_I and x_i $(I, i = 1, 2, 3)$, and the displacement of a material point is denoted by $u_i = x_i - X_I$. The deformation gradient tensor is $\boldsymbol{F} = \nabla_X \mathbf{u} + \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity tensor and ∇_X denotes gradient operator in the reference configuration. The right Cauchy–Green deformation tensor is given as $\boldsymbol{C} = \boldsymbol{F}^T \boldsymbol{F}$ with the Jacobian $J = \det(\boldsymbol{F}) > 0$. The first and second Piola–Kirchhoff (PK) stresses are respectively expressed in terms of the Cauchy stress $\boldsymbol{\sigma}$ as [30]:

$$\boldsymbol{P} = J\boldsymbol{\sigma} \cdot \boldsymbol{F}^{-T} \quad \text{and} \quad \boldsymbol{S} = J\boldsymbol{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{F}^{-T} \tag{1}$$

The equilibrium equation for the coupled dynamic and electric problem at time t is expressed in the reference configuration as:

$$\frac{\partial^t P^m_{iJ}}{\partial X_J} + \frac{\partial^t P^M_{iJ}}{\partial X_J} + \rho_0 b^m_i = \rho_0 \ddot{u}_i \quad \text{in} \quad {}^0\Omega \tag{2}$$

where P^m_{iJ} and P^M_{iJ} are respectively the components of the first Piola–Kirchhoff (PK) stress tensor due to the mechanical and electrical fields, ρ_0 is the density in the reference configuration and b^m_i is the mechanical body force per unit mass.