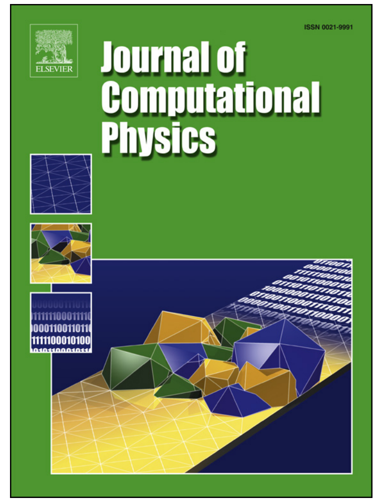

Invariant-region-preserving DG methods for multi-dimensional hyperbolic conservation law systems, with an application to compressible Euler equations

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INVARIANT-REGION-PRESERVING DG METHODS FOR MULTI-DIMENSIONAL HYPERBOLIC CONSERVATION LAW SYSTEMS, WITH AN APPLICATION TO COMPRESSIBLE EULER EQUATIONS

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Abstract. An invariant-region-preserving (IRP) limiter for multi-dimensional hyperbolic conservation law systems is introduced, as long as the system admits a global invariant region which is a convex set in the phase space. It is shown that the order of approximation accuracy is not destroyed by the IRP limiter, provided the cell average is away from the boundary of the convex set. Moreover, this limiter is explicit, and easy for computer implementation. A generic algorithm incorporating the IRP limiter is presented for high order finite volume type schemes. For arbitrarily high order discontinuous Galerkin (DG) schemes to hyperbolic conservation law systems, sufficient conditions are obtained for cell averages to remain in the invariant region provided the projected one-dimensional system shares the same invariant region as the full multi-dimensional hyperbolic system does. The general results are then applied to both one and two dimensional compressible Euler equations so to obtain high order IRP DG schemes. Numerical experiments are provided to validate the proven properties of the IRP limiter and the performance of IRP DG schemes for compressible Euler equations.

1. Introduction

The multi-dimensional hyperbolic conservation law systems are given by

$$\partial_t \mathbf{w} + \sum_{j=1}^{d} \partial_{x_j} F_j(\mathbf{w}) = 0, \quad x \in \mathbb{R}^d, \quad t > 0 \tag{1.1}$$

with the unknown vector $\mathbf{w} \in \mathbb{R}^l$ and the flux function $F_j(\mathbf{w}) \in \mathbb{R}^l$ for $j = 1, \cdots d$. We consider the initial value problem for system (1.1) with the initial data $\mathbf{w}(x, 0) = \mathbf{w}_0(x)$. For simplicity, we take periodic or compactly supported boundary conditions.

It is well known that entropy inequalities should be considered for general hyperbolic conservation laws so to single out the physically relevant solution among many weak solutions (see, e.g., [18]). In application problems, the pointwise range of solutions may be known from physical considerations, instead of total entropy. For scalar conservation laws, the entropy solution satisfies a strict maximum principle. For hyperbolic conservation law systems, the notion of maximum principle does not apply and must

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