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Short note

Affordable shock-stable item for Godunov-type schemes against carbuncle phenomenon

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A B S T R A C T

The work proposes a simple, efficient and unified approach to combat carbuncle phenomenon. The basic idea of this approach is to construct an affordable shock-stable item into numerical flux. This item is acquired by comparing three-wave Roe solver and two-wave HLL solver based on Roe linearization and the projection of difference term. It is consistent with shear viscosity and meantime restrained by the pressure-based sensing function for shear layer. The proposed enhancement is easy to implement and apply within Roe, HLLEM, HLLC, AUSM+, etc. Several well-known cases illuminate its shock robustness and potential application to hypersonic flows.

1. Affordable shock-stable item

Shock-capturing or upwind schemes in Computational Fluid Dynamics (CFD) have prevailed and matured in the computations of compressible flows. However, low-diffusion upwind schemes (e.g. Roe [1], HLLEM [2], HLLC [3], AUSM+ [4], etc.) are more or less vulnerable to shock instability or carbuncle phenomenon for hypersonic speeds. To alleviate this dilemma, many investigators have made tremendous efforts. For instance, Quirk [5] proposed a hybrid strategy using high-dissipative schemes near shock waves and relative low-dissipative schemes elsewhere, which has been further developed by Nishikawa [6], Kim [7], etc. Liou [8] confirmed that the pressure difference term of mass flux has a great influence on shock instability, and this idea has promoted many shock-stable schemes [9]. Chen et al. [10] developed a normal velocity reconstruction (NVR) procedure with transverse information to suppress shock instability. Rodionov [11] suggested an artificial viscosity (AV) associated with viscous flux based on von Neumann and Richtmyer artificial viscosity. In the current work, the motivation is (1) to explain what kind of numerical viscosity is conducive to suppressing shock instability; (2) to propose another simple, efficient and unified approach to improve shock robustness against carbuncle instability.

The Euler equations in the x-direction are expressed in conservation form ($\partial_t \boldsymbol{Q} + \partial_x \boldsymbol{F}(\boldsymbol{Q}) = 0$) as:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho e\end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix}\rho U\\ \rho U u + p n_x\\ \rho U v + p n_y\\ \rho U h\end{pmatrix} = 0 \tag{1}$$

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with density ρ, velocity $\boldsymbol{u} = (u, v)^T$, pressure p, total specific energy e, total enthalpy h, the unit normal vector $\boldsymbol{n} = (n_x, n_y)^T$ and the normal velocity $U = n_x u + n_y v$. Based on the prefect law with the specific heat ratio $\gamma = 1.4$, the following relationship is established

$$a = \sqrt{\frac{\gamma p}{\rho}}, \qquad e = \frac{p}{(\gamma - 1)\rho} + \frac{\|\boldsymbol{u}\|^2}{2}, \qquad h = e + \frac{p}{\rho} \tag{2}$$

This system is numerically solved by a conservative method

$$\boldsymbol{Q}_i^{n+1} = \boldsymbol{Q}_i^n - \frac{\Delta t}{\Delta x}[\boldsymbol{F}_{i+1/2} - \boldsymbol{F}_{i-1/2}] \tag{3}$$

with the numerical flux $\boldsymbol{F}_{i+1/2}$ evaluated by Riemann solvers, such as Roe [1], HLL [12], HLLEM [2], HLLC [3], AUSM+ [4], etc. In the current study, three-wave Roe solver and two-wave HLL solver are presented. The famous Roe solver [1] is given as:

$$\boldsymbol{F}_{1/2}^{Roe}(\boldsymbol{Q}_L, \boldsymbol{Q}_R) = \frac{\boldsymbol{F}(\boldsymbol{Q}_L) + \boldsymbol{F}(\boldsymbol{Q}_R)}{2} - \frac{1}{2}|\tilde{\boldsymbol{A}}|\Delta \boldsymbol{Q} \tag{4}$$

The subscripts "L" and "R" refer to the left and right states respectively, determined by reconstruction methods. The difference between the left and right states is shown with the operator $\Delta(\cdot) = (\cdot)_R - (\cdot)_L$. The superscript tilde "$\tilde{}$" stands for Roe average. Roe-averaged matrix $\tilde{\boldsymbol{A}}$ is similar to the convective flux Jacobian matrix $\boldsymbol{A} = \partial \boldsymbol{F}/\partial \boldsymbol{Q}$, while flow variables are replaced by Roe-averaged variables [1]. Despite high resolution of contact and shear waves, Roe is known to be vulnerable to carbuncle instability.

Two-wave HLL solver [12] is very robust against carbuncle instability but lacks contact and shear waves. It can be defined as

$$\begin{aligned} &\boldsymbol{F}_{1/2}^{HLL}(\boldsymbol{Q}_L, \boldsymbol{Q}_R) = \frac{S_R^+ \cdot \boldsymbol{F}(\boldsymbol{Q}_L) - S_L^- \cdot \boldsymbol{F}(\boldsymbol{Q}_R)}{S_R^+ - S_L^-} + \frac{S_R^+ S_L^-}{S_R^+ - S_L^-}\Delta \boldsymbol{Q} \\ &S_R^+ = \max(0, U_R + a_R, \tilde{U} + \tilde{a}), \qquad S_L^- = \min(0, U_L - a_L, \tilde{U} - \tilde{a}) \end{aligned} \tag{5}$$

Then, we compare Roe with HLL to attain the affordable shock-stable item via the following key steps.

Step 1: Project the difference term $\Delta \boldsymbol{Q}$ onto the right eigenvector $\boldsymbol{r}_k$

$$\Delta \boldsymbol{Q} = \boldsymbol{Q}_R - \boldsymbol{Q}_L = \sum_{k=1}^{4} \Delta l_k \cdot \boldsymbol{r}_k \tag{6}$$

Wave strengths Δl_k are given as

$$\Delta l_1 = \frac{\Delta p - \tilde{\rho}\tilde{a}\Delta U}{2\tilde{a}^2}, \qquad \Delta l_2 = \Delta \rho - \frac{\Delta p}{\tilde{a}^2}, \qquad \Delta l_3 = \frac{\Delta p + \tilde{\rho}\tilde{a}\Delta U}{2\tilde{a}^2}, \qquad \Delta l_4 = \tilde{\rho}\Delta V \tag{7}$$

with the transverse velocity is $V = -n_y u + n_x v$. Δl_1 is left-moving acoustic wave strength, Δl_2 is entropy wave strength, Δl_3 is right-moving acoustic wave strength, Δl_4 is shear wave strength.

Right eigenvector $\boldsymbol{r}_k$ is as follows

$$\boldsymbol{r}_1 = \begin{bmatrix} 1 \\ \tilde{u} - \tilde{a}n_x \\ \tilde{v} - \tilde{a}n_y \\ \tilde{h} - \tilde{a}\tilde{U} \end{bmatrix}, \qquad \boldsymbol{r}_2 = \begin{bmatrix} 1 \\ \tilde{u} \\ \tilde{v} \\ (\tilde{u}^2 + \tilde{v}^2)/2 \end{bmatrix}, \qquad \boldsymbol{r}_3 = \begin{bmatrix} 1 \\ \tilde{u} + \tilde{a}n_x \\ \tilde{v} + \tilde{a}n_y \\ \tilde{h} + \tilde{a}\tilde{U} \end{bmatrix}, \qquad \boldsymbol{r}_4 = \begin{bmatrix} 0 \\ -n_y \\ n_x \\ \tilde{V} \end{bmatrix} \tag{8}$$

with left-moving acoustic wave $\boldsymbol{r}_1$, entropy wave $\boldsymbol{r}_2$, right-moving acoustic wave $\boldsymbol{r}_3$ and shear wave $\boldsymbol{r}_4$.

Step 2: Use Roe linearization to obtain wave speed

Roe solver (Eq. (4)) can be further converted into

$$\begin{aligned} &F_{1/2}^{Roe} = \frac{F_L + F_R}{2} - \frac{1}{2}\sum_{k=1}^{4} \Delta l_k \cdot \boldsymbol{r}_k \cdot |\lambda_k| \\ &\lambda_1 = \tilde{U} - \tilde{a}, \qquad \lambda_2 = \tilde{U}, \qquad \lambda_3 = \tilde{U} + \tilde{a}, \qquad \lambda_4 = \tilde{U} \end{aligned} \tag{9}$$

where λ_1 is left-moving acoustic wave speed, λ_2 is entropy wave speed, λ_3 is right-moving acoustic wave speed, λ_4 is shear wave speed.