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A fast numerical framework to compute acoustic scattering by poroelastic plates of arbitrary geometry

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ABSTRACT

We present a fast numerical framework for the computation of acoustic scattering by poroelastic plates of arbitrary geometries. A boundary element method, BEM, is applied to solve the Helmholtz equation subjected to boundary conditions related to structural vibrations. This analysis is performed by rewriting the BEM boundary conditions in terms of a modal basis of the poroelastic plate which is computed by the finite element method, FEM. The current formulation allows a direct solution of the fully coupled fluid-structure interaction problem. In order to accelerate the solution of the large dense linear systems from the BEM formulation in three-dimensional problems, a wideband adaptive multi-level fast multipole method, FMM, is employed. A parametric study is carried out for the trailing-edge scattering of sample acoustic sources, representative of either uncorrelated turbulent eddies or a non-compact turbulent jet. Firstly, the noise scattering by a compact quadrupole source is analyzed for low and high frequencies for square and trapezoidal plates. Results show that geometric features such as trailing-edge sweep and serrations are very effective in the reduction of noise scattering. Moreover, it is shown that finite elastic plates are more effective in reducing the scattered noise at higher frequencies. On the other hand, porosity is more effective in reducing the radiated sound for lower frequencies. Results demonstrate that elasticity and porosity can be combined with trailing-edge sweep and serrations to reduce the scattered noise at a broader range of frequencies for poroelastic plates.

1. Introduction

The convection of turbulent structures past an airfoil trailing edge leads to noise generation through a scattering mechanism [1]. In this context, the unsteady flow generates surface pressure fluctuations which are scattered and propagated to the far field [2]. Several problems in engineering involve noise scattering by solid surfaces; for example, the development of turbulent boundary layers along an airfoil finds application in the noise generated by wings, rotorcrafts, wind turbines, fans and high-lift devices. It is thus of interest to predict and understand sound generation taking place at the trailing edge, such that one may propose wing modifications capable of reducing the radiated sound.

Several species of owls can reduce, for some ranges of frequencies, the noise scattering during flight and this characteristic is responsible for their reputation as silent predators. This noise reduction is associated to specific properties of

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the owl's wings, which include leading-edge bristles and permeable-elastic trailing-edge fringes. In this case, fluid-structure interaction plays an important role since acoustic pressure fluctuations incident on the elastic surface lead to the propagation of structural waves that scatter on the surface discontinuities (edges, corners and rigid-elastic junctions) generating an interference in the acoustic field. Moreover, the structural waves also modify the acoustic signature along the elastic surface.

Recently, Jaworski & Peake [3] performed a theoretical analysis and demonstrated that the noise scattered by a trailing edge can be mitigated by the application of edge elasticity and porosity for certain parameter ranges. In their theoretical analysis, the assumption of a semi-infinite poroelastic edge is made and, therefore, its application is well suited for trailing-edge scattering in the high frequency regime. More recently, Ayton [4] and Pimenta et al. [5] presented theoretical and numerical investigations, respectively, of the scattered sound by a finite rigid plate and a NACA0012 airfoil with poroelastic plate extensions. Both these analyses assume that the poroelastic extensions are two-dimensional, *i.e.*, effects of scattering on lateral edges and corners are not considered.

In order to investigate the effects of low and high frequency scattering along poroelastic trailing edges, aerodynamic configurations with finite chord and span need to be considered. Finite plates are susceptible to structural resonance [6–8] and secondary diffraction by leading [9] and lateral edges and corners [10]. Cavalieri et al. [11,12] developed a novel formulation for the study of acoustic scattering by poroelastic plates with finite chord and infinite span.

Most investigations of acoustic scattering involving poroelastic finite edges assume either infinite span or rectangular plates. It is known that swept trailing edges should reduce the efficiency of the scattering mechanism from a turbulent eddy in the proximity of the edge [1]. Recently, Piantanida et al. [13] showed that jet-installation noise, generated from the interaction of a turbulent jet with a neighboring wing, can be substantially reduced by increasing the sweep angle of a rigid trailing edge. Serrations are also employed to reduce the noise scattering by rigid trailing edges [14]. For the latter case, numerical simulations and experiments have been employed to investigate the flow and acoustic radiation by airfoils and plates with trailing-edge serrations [15–17].

In the current work, we present a fast numerical framework to study the acoustic scattering by fully three-dimensional poroelastic plates with finite chord and span. To investigate the effects of noise scattering on the 3D plates of arbitrary geometry, we employ a theoretical formulation similar to that proposed by Ref. [12]. The numerical methodology combines well-established numerical techniques such as the finite element method, FEM, the boundary element method, BEM, and the fast multipole method, FMM.

The 3D wideband adaptive fast multipole boundary element method developed in Refs. [18,19] is adapted to solve the acoustic problem due to an acoustic source in the proximity of a poroelastic edge. The solution of the structural problem for an elastic plate is obtained by a generalized formulation which provides a modal basis computed by the finite element method. The structural and acoustic problems are coupled by the linearized Euler equation, which provides the boundary conditions relating the plate displacement to the acoustic pressure. The current work shows a detailed study about the acoustic scattering by square and trapezoidal poroelastic plates excited by compact and non-compact noise sources. A thorough investigation of the effects of elasticity, porosity and their combination is provided for plates including serrations and for a jet-plate installation problem. A validation analysis of the present numerical framework is also presented comparing 2D and 3D solutions of acoustic scattering by poroelastic edges. With the present framework, one can thus evaluate the acoustic scattering in a wide range of frequencies by plates with one or several of the noise-reduction mechanisms referred to above.

2. Mathematical model and numerical framework

The model problem of acoustic scattering by a three-dimensional poroelastic plate is shown schematically in Fig. 1. This figure shows a sound source $\mathcal{S}$ placed on the vicinity of one of the edges of the poroelastic plate. The total sound field, which is composed by the incident and scattered sounds, is computed at the observer position r. Although the figure depicts a rectangular plate with one clamped leading edge and free trailing and lateral edges, it is important to mention that the current formulation can be applied to other combinations of structural boundary conditions and plate geometries.

The sound source is placed in the proximity of the trailing edge and the near pressure field is scattered by the plate, radiating to the far-field [20]. The scattered sound radiates as a distribution of surface dipole and monopole sources according to the Helmholtz integral equation. At the same time, the incident acoustic field excites structural waves along the plate, which propagate along the elastic surface. These waves hit the clamped leading edge and the lateral edges and corners of the plate, being reflected. Secondary acoustic diffraction takes place at the leading and lateral edges of the plate due to the finite dimensions of the plate and due to the impingement of elastic waves.

Assuming a harmonic time dependence $\exp\left(-\mathrm{i}\tilde{\omega}\tilde{t}\right)$, we compute the scattered sound solving the non-dimensional Helmholtz equation as

$$\nabla^2 p + k_0^2 p = -\mathcal{S}. \tag{1}$$

Here, $\mathcal{S}$ is the acoustic source function, and k_0 is the Helmholtz number calculated based on the plate chord $\mathrm{He} = k_0 = \tilde{k}_0\tilde{L}$. The overhead tildes indicate dimensional quantities and $\tilde{k}_0$ is the acoustic wavenumber given as $\tilde{\omega}/\tilde{c}_0$ for angular frequency $\tilde{\omega}$ and speed of sound $\tilde{c}_0$. The plate chord $\tilde{L}$ is employed as the characteristic length scale in the current work and all the equations are written using the non-dimensional form unless otherwise stated. More details about the nondimensionalization procedure of the present equations can be found in [12].