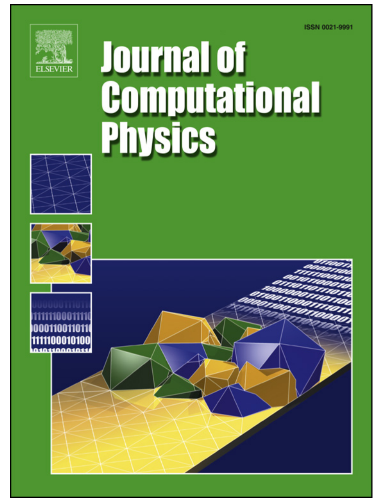

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Finite Element Discretization of a Stokes-like Model Arising in Plasma Physics

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Abstract

We consider a time-dependent diffusion-reaction model for two vector unknowns, satisfying a divergence-free constraint, and the associated scalar Lagrange multiplier. The motivation for studying such a model is provided by a plasma physics problem arising in the modeling of nuclear fusion devices (Braginskii equations), where the two vector unknowns represent ion and electron velocities, the scalar unknown is the electrostatic potential and the divergence-free constraint reflects the physical assumption of quasi-neutrality. We first recast the problem in a form reminiscent of the standard Stokes problem, which allows us to recognize the importance of using a compatible discretization for the vector and scalar unknowns, then propose and analyze a stable finite element formulation. Following this, we address some peculiar geometrical aspects of the model, showing how they can be naturally dealt with within our formulation, and finally discuss a solution procedure for the resulting linear system based on the classical Uzawa algorithm. Some numerical experiments complete the paper.

Keywords: Finite elements, inf-sup stable discretization, Braginskii equations, quasi-neutrality condition, plasma physics, tokamak modeling
2010 MSC: 65M60, 76D07, 76M10, 76W05, 76X05, 82D10

1. Introduction

The magnetic confinement approach to nuclear fusion for civil applications relies on the construction of large toroidal devices where a hydrogen plasma is heated while being confined by a strong magnetic field. In order to obtain the plasma ignition, three simultaneous conditions must be fulfilled: high temperature, high density and long confinement time. Ensuring these conditions has proven to be a major technological challenge, which must be supported by a deep physical understanding of the involved processes; in this context, an important role is played by the use of numerical models.

In this paper, we are interested in fluid models which are used to describe the heat and particle fluxes occurring in the peripheral region of the confined plasma, the so-called Scrape-off Layer (SOL) [1]. A suitable

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