



Fourier analysis and evaluation of DG, FD and compact difference methods for conservation laws

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ARTICLE INFO

Article history:

Received 19 January 2018

Received in revised form 29 April 2018

Accepted 11 July 2018

Available online 17 July 2018

Keywords:

Discontinuous Galerkin method

Compact difference

Finite difference

Dispersion-dissipation analysis

Combined-mode analysis

Implicit LES

ABSTRACT

Large eddy simulation (LES) has been increasingly used to tackle vortex-dominated turbulent flows. In LES, the quality of the simulation results hinges upon the quality of the numerical discretizations in both space and time. It is in this context we perform a Fourier analysis of several popular methods in LES including the discontinuous Galerkin (DG), finite difference (FD), and compact difference (CD) methods. We begin by reviewing the semi-discrete versions of all methods under consideration, followed by a fully-discrete analysis with explicit Runge–Kutta (RK) time integration schemes. In this regard, we are able to unravel the true dispersion/dissipation behavior of DG and Runge–Kutta DG (RKDG) schemes for the entire wavenumber range using a *combined-mode* analysis. In this approach, we take into account all eigenmodes in DG and RKDG schemes. The physical-mode is verified to be a good approximation for the asymptotic behavior of these DG schemes in the low wavenumber range. After that, we proceed to compare the DG, FD, and CD methods in dispersion and dissipation properties. Numerical tests are conducted using the linear advection equation to verify the analysis. In comparing different methods, it is found that the overall numerical dissipation strongly depends on the time step. Compact difference (CD) and central FD schemes, in some particular settings, can have more numerical dissipation than the DG scheme with an upwind flux. This claim is then verified through a numerical test using the Burgers' equation.

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1. Introduction

According to NASA's 2030 Vision on CFD [1], scale-resolving simulations such as large eddy simulation (LES), will be increasingly used to compute challenging vortex-dominated turbulent flow problems. Multiple international workshops on high-order CFD methods [2] have conclusively demonstrated the advantage of high-order methods over 1st and 2nd order ones in accuracy/efficiency for such scale-resolving simulations due to their lower dispersion and dissipation errors. Interested readers can refer to several review articles on high-order methods [3–6].

It was previously shown that some upwind-biased FD schemes are too dissipative [7–10] to be a viable numerical approach for LES. Unfortunately, some researchers extrapolated from this to dismiss any kind of “upwinding” in numerical methods including DG-type methods, and argue for non-dissipative methods such as central FD schemes for LES. However, time integration approaches such as the Runge–Kutta (RK) scheme to introduce numerical dissipation. It is therefore very important to analyze the fully-discretized versions to quantify the amount of dispersion and dissipation errors. The main

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objective of the present study is to compare the fully-discretized DG, FD and CD methods, and assess their performance for model equations.

In the context of LES, there is physical dissipation associated with the molecular viscosity. In addition, there is dissipation associated with the sub-grid-scale (SGS) stress, and finally there is numerical dissipation associated with the selected numerical method. The actual SGS stress obviously depends on the quality of the SGS model. Sometimes, the SGS stress provided by a model does not really correlate well with the physical SGS stress [11]. In this case, the role of the SGS model is to stabilize the simulation. For a central FD scheme, a dissipative SGS model is essential in achieving a successful simulation. While for dissipative methods such as the DG, or CD scheme with spatial filtering, it is often not necessary to include a SGS model since the numerical dissipation is sufficient to stabilize the simulation. There have been overwhelming evidence which shows that adding a SGS model can be detrimental to the solution quality [12,10,13,14] for dissipative methods. In practice, implicit LES (ILES) has been shown to perform very well for a variety of flow problems [13,15–19].

In order to assess the dispersion/dissipation characteristics and resolution of a numerical scheme, Fourier analysis [20] is often utilized either in a semi-discrete [21–25] or fully discrete setting [26–29]. In our present work, we start with a review of semi-discrete schemes, and then proceed to analyze the fully discrete schemes assuming a periodic boundary condition for the sake of simplicity. For high-order DG-type methods, most of the previous work studied the behavior of high-order schemes based on what is called the physical-mode [21,22,24] defined as the one that approximates the exact dispersion relation for a range of wavenumbers while regarding other modes as spurious. Recently, Moura et al. [27] provided new interpretations on the role of spurious or secondary modes. In their work, these modes are replicates of the physical-mode along the wavenumber axis and they improve the accuracy of the scheme. Vanharen et al. [29] concluded that after a large number of iterations, high-order schemes behave in dispersion and dissipation according to the physical-mode asymptotically, for wavenumbers less than π . Nevertheless, the complete behavior of DG-type high-order schemes in dispersion and dissipation based on all eigenmodes has not been studied before. In this paper, we provide a first attempt to achieve this goal by introducing the *combined-mode* analysis.

Whilst there exists abundant work on the analysis of both high-order and low-order schemes or classical finite difference/finite-volume schemes, little attention was given to comparing the fully discretized DG, FD, and CD schemes of the same order of accuracy. The DG method, originally introduced by Reed and Hill [30] to solve the neutron transport equation, is chosen in this study as a representative of the high-order polynomial-based methods capable of handling unstructured grids including the spectral difference (SD) [31], and the flux reconstruction (FR) or correction procedure via reconstruction (CPR) methods [32]. LaSaint and Raviart [33] performed an error analysis for the DG method. It was then further developed for convection-dominated problems and fluid dynamics by many researches, see for example [34–40] and the references therein. In addition, the CD method of Lele [41] is also analyzed for comparison purposes. This method was further developed by Gaitonde et al. [42] and Visbal et al. [43], and applied successfully to perform ILES by Visbal et al. [44,12] and Rizzetta et al. [15]. Recently, a comparative study of the suitability of the method for ILES versus SGS was conducted by Garman et al. [14] and San has utilized the method for an analysis of low-pass filters for the approximate deconvolution closure [45]. Finally, we have also included central and upwind-biased FD schemes in the present comparison to illustrate the performance of a fundamental classical method.

The class of FD schemes utilized in this work is usually referred to as standard FD schemes. Another interesting class of FD schemes is the optimized Dispersion-Relation-Preserving (DRP) [46–49] schemes for computational aeroacoustics (CAA). These schemes minimize the dispersion error in different norms for some low wavenumber range by changing the coefficients of the standard (FD) scheme of the same stencil. For a comparison of the relative efficiencies of the DRP schemes versus standard FD ones, the reader can consult the following references [50–52].

Unlike DG-type methods, CD and FD schemes need to change the discretization stencil near boundaries, which may suffer from instability. This problem can be remedied using spatial filters and/or numerical boundary schemes (NBS) [53]. Another interesting approach to ensure stability is to use schemes that satisfy the Summation-by-Parts property (SBP) see for example [54,55] and the references therein. It is also worth noting that connections between DG-type methods and the SBP property have been studied in [56,57]. In this article, we focus on the interior schemes with a periodic boundary condition for the sake of simplicity.

For time integration, we focus on the explicit RK [58–60] method to demonstrate the importance of analyzing the fully discrete version. RK schemes are easily incorporated with high-order methods such as the RKDG [35] due to their ease of implementation and parallelization.

In the present study, we first review the analysis of semi-discrete schemes, followed by a fully-discrete analysis for three classes of methods, namely, the DG, FD and CD methods. We clarify the relative efficiency and robustness of each method in terms of wave propagation properties. In addition, using the *combined-mode* analysis, we study the true dispersion/dissipation properties of DG and RKDG schemes. It is verified that, at least in the low wavenumber range, the physical-mode (defined by Hu et al. [21]) can serve as a good approximation for the complete behavior of a high-order scheme.

This paper is organized as follows. Section 2 introduces the basic formulations of all numerical methods under consideration in the present study. After that, we present a semi-discrete analysis followed by a fully discrete one in Section 3. Section 4 presents the comparison of dispersion/dissipation behavior of the DG, FD and CD schemes coupled with RK schemes. Numerical verifications and test cases are presented in Section 5. Finally, conclusions are summarized in Section 6.

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