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Non-local multi-continua upscaling for flows in heterogeneous fractured media

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ABSTRACT

In this paper, we propose a rigorous and accurate non-local (in the oversampled region) upscaling framework based on some recently developed multiscale methods [10]. Our proposed method consists of identifying multi-continua parameters via local basis functions and constructing non-local (in the oversampled region) transfer and effective properties. To achieve this, we significantly modify our recent work proposed within Generalized Multiscale Finite Element Method (GMsFEM) in [10] and derive appropriate local problems in oversampled regions once we identify important modes representing each continuum. We use piecewise constant functions in each fracture network and in the matrix to write an upscaled equation. Thus, the resulting upscaled equation is of minimal size and the unknowns are average pressures in the fractures and the matrix. Note that the use of non-local upscaled model for porous media flows is not new, e.g., in [14], the authors derive non-local approach. Our main contribution is identifying appropriate local problems together with local spectral modes to represent each continuum. The model problem for fractures assumes that one can identify fracture networks. The resulting non-local equation (restricted to the oversampling region, which is several times larger compared to the target coarse block) has the same form as [14] with much smaller local regions. We present numerical results, which show that the proposed approach can provide good accuracy.

1. Introduction

Because of the level of details in geological formations, some type of coarsening or upscaling is typically performed. In upscaling methods, media properties are upscaled and effective properties are computed for each coarse block [2,18,12,15,4,41,17]. Computing effective properties involves solving local problems and equating the averages of local integrated quantities. For example, computing upscaled permeabilities in reservoir simulation is typically based on equating average fluxes between the local fine-grid solves and the coarse-grid solves. This equality allows computing the effective permeability fields.

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In a more general upscaling setup, a multiple continua approach [8] is needed. In this approach, several effective properties are computed for each coarse block in addition to modeling the transfer terms. The computations involve evaluating both effective properties and transfer coefficients between different continua, and are performed locally.

Similar to upscaling methods, many authors have recently studied multiscale methods. In multiscale methods [27,25,30,33,21,11,28,3,29,35,31,34,20,7,24,26,9], instead of computing the effective properties, one computes multiscale basis functions. For single-phase upscaling, a multiscale basis function for each coarse node is computed via local solutions. These basis functions are further coupled via a global formulation of the equation. This approach is implemented within Multiscale Finite Element Method (MsFEM) and other multiscale methods [27,28,20,7,1,37,36,5,6].

To generalize this approach to more complex heterogeneities and the multi-continua case, Generalized Multiscale Finite Element Method (GMsFEM) is proposed [20,7,9,10]. GMsFEM proposes a systematic approach to compute multiple basis functions. This approach starts with a space of snapshots, where one performs local spectral decomposition to compute multiscale basis functions. Adaptivity can be used to select basis functions in different regions. Each multiscale basis function represents a continuum as discussed in [8] and there is no need for coupling terms between these continua. The basis functions for each continuum are automatically identified.

The GMsFEM approach has been used jointly with localization ideas in [10], where the authors propose Constraint Energy Minimizing GMsFEM. In this approach, oversampling regions are used to compute the multiscale basis functions. This construction takes into account spectral basis functions to localize the computations. The localization is restricted to $\log(H)$ layers and depends on the contrast, which can be reduced using snapshot functions. Moreover, it was shown that the approach converges independent of the contrast and the convergence is linear with respect to the coarse mesh size. More precisely, the convergence is proportional to $H/\Lambda^{1/2}$, where Λ is the smallest eigenvalue that the corresponding eigenfunction is not included in the coarse space. Note that basis functions associated to fractures correspond to very small eigenvalues. The goal of this paper is to modify this framework in an appropriate way that is more suitable for flow-based upscaling and re-cast it as non-local upscaling.

In this paper, in order to modify the multiscale approach presented in [10], we first assume that one knows each separate fracture network within a coarse-grid block. This is one of the drawbacks of our method; however, such cases occur in many applications. Next, we follow a general concept of spectral basis functions and simply define constant functions in each fracture network and the matrix. Because the fracture has zero width, this procedure needs to be carefully formulated, which is done in the paper.

Secondly, we solve local problems in the oversampled region subject to the constraint that the local problems vanish in fractures and the matrix. This constraint is important for the localization. The local problems formulated for each continuum (either fracture network or the matrix phase) simply minimize the local energy subject to the constraint that the local solution "vanishes" in other continua except the one for which it is formulated for. More precisely, for the continuum i in a block K, we minimize local oversampled solution such that it is orthogonal to all continua except i and an appropriate inner product with the continuum i is 1.

It is important to note that the localization will not be possible if we did not identify and separate each fracture network. This is due to the fact that the effects of fractures are not localizable and are global as it is well known.

Next, we use these local solutions to compute the upscaled equation. Because the local calculations are done in an oversampled domain, the transmissibilities are non-local and extend to the oversampled region, which is $\log(H)$ layers around the target coarse block. Our coarse-grid equations have a similar form to those [14]; however, our derivations significantly differ. In [14], the authors derive non-local Darcy equation, where the connectivity between far regions is taken into consideration. In our work, local problems in addition to multi-continua as well as localization are employed. Moreover, we show that one can obtain an accurate solution independent of the contrast and the mesh size. The resulting upscaled equation is written in a discrete form as

$$\sum_{j,n} T_{mn}^{i,j}(u_n^{(j)} - u_m^{(i)}) = q_m^{(i)},$$

where $T_{mn}^{i,j}$ are nonlocal transmissibilities for different continua m and n, and i, j correspond to different coarse blocks. The transmissibilities $T_{mn}^{i,j}$ are defined in oversampled regions and the non-local dependence of them are investigated. Note that the proposed approach modifies the framework developed in [10] to derive the non-local multiple continuum upscaled models.

Non-local upscaling is not new in porous media [13,32,40,19,22,39,16], particularly for transport equations. However, even in elliptic equations, one can obtain non-local upscaling results. Our proposed method is motivated by the work of Jenny et al., [14], where they derive non-local upscaled models. A recent paper [23] is also worth mentioning, where the authors derive non-local upscaling for problems without high contrast. The upscaling for flows in fractured media requires multi-continua. Thus, to avoid the global upscaling, one needs to take into account the fractures separately and localize their effects. In all these papers, the global formulations of the resulting macroscopic equations are the same, and the main difference lies in the computation of upscaled quantities. In this regard, our approach differs from existing works in the literature and addresses a general case of problems with high contrast and multiple scales.

In the paper, some numerical results are presented, where we compare our proposed upscaled model and the fine-grid models. Both averages and downscaled quantities are studied. Our numerical results show that one can achieve a good