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Second-order implicit-explicit total variation diminishing schemes for the Euler system in the low Mach regime

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ABSTRACT

In this work, we consider the development of implicit-explicit total variation diminishing (TVD) methods (also termed SSP: strong stability preserving) for the compressible isentropic Euler system in the low Mach number regime. The proposed scheme is asymptotically stable with a CFL condition independent of the Mach number. In addition, it degenerates, in the low Mach number regime, to a consistent discretization of the incompressible system. Since it has been proved that implicit schemes of order higher than one cannot be TVD (SSP) [30], we construct a new paradigm of implicit time integrators by coupling first-order in time schemes with second-order ones in the same spirit as highly accurate shock-capturing TVD methods in space. For this particular class of schemes, the TVD property is first proved on a linear model advection equation and then extended to the isentropic Euler case. The result is a method which interpolates from the first- to the second-order both in space and time. It preserves the monotonicity of the solution, and is highly accurate for all choices of the Mach number. Moreover, the time step is only restricted by the non-stiff part of the system. We finally show, thanks to one- and two-dimensional test cases, that the method indeed possesses the claimed properties.

1. Introduction

The analysis [44,45,66,2,48,1] and the development of numerical methods [38,34,60,72,46,13,32,56,52,35,31,55,53,17,20,18,16,33,10,29,43,21,11,22] for the passage from compressible to incompressible gas dynamics has been and still is a very active field of research. The compressible Euler equations, which describe conservation of density, momentum and energy in a fluid flow, become stiff when the Mach number tends to zero.

In this case, the velocity of the pressure waves is much greater than that of the gas. Thus, a standard model approximation consists in replacing the density conservation equation by a constraint on the velocity divergence, set consequently equal to zero. In addition, using this constraint into the momentum equation gives an elliptic equation for the pressure. We refer to that situation as the incompressible Euler model, which is used to describe many different flow conditions. However, there are situations in which the Mach number may be small in some parts of the domain and large in others,

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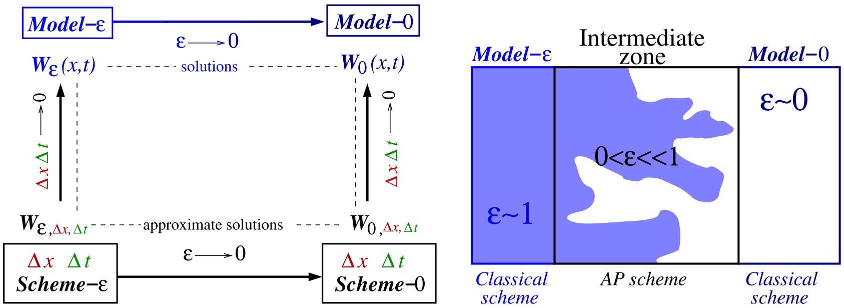

Fig. 1. Left: Asymptotic Preserving diagram. The schemes $W_{\varepsilon,\Delta x,\Delta t}$ and $W_{0,\Delta x,\Delta t}$ are consistent discretizations of the models $W_\varepsilon(x,t)$ and $W_0(x,t)$. An Asymptotic Preserving method is such that when $\varepsilon \to 0$, the scheme $W_{\varepsilon,\Delta x,\Delta t}$ automatically becomes the scheme $W_{0,\Delta x,\Delta t}$. Right: An example in which different regimes coexist. While a standard scheme for the $\mathcal{M}_\varepsilon$ model or the $\mathcal{M}_0$ model may only work for large values of ε or when $\varepsilon = 0$, the Asymptotic Preserving scheme works in all regimes and in intermediate regions it is the only one that should be employed.

or may strongly change in time. In these cases, one should deal with the coupling of incompressible and compressible regions whose shapes change in time. From the numerical point of view, this causes many difficulties since standard domain decomposition techniques, which couple the solution of the compressible equations with the solution of the incompressible system, may be difficult to use (see [5]). Thus, one solution consists in solving the more complete compressible Euler system in the stiff regime. As a consequence, this introduces strong drawbacks when the Mach number becomes small. The first one is related to the fact that classical explicit schemes for the compressible Euler system are not uniformly stable. Their CFL condition is inversely proportional to the Mach number. This causes severe time step limitations in low Mach number regimes. The second drawback is a consistency problem. Indeed, it is well known (see for instance [31,56,20,21]) that explicit Roe type solvers are not consistent in the low Mach number limit. In particular, they fail to describe the limit pressure.

From the physical point of view, it is important to understand that two different situations are possible. In the first situation, sound waves play an important role in the physical problem. Then, even if their velocity is much greater than the one of the fluid, it is crucial to accurately capture these sound waves in this situation. Consequently, the time step of the discretization must be of the order of the Mach number. However, even in these situations, the consistency problems of low Mach regimes should be solved. This problem can be treated by preconditioning and/or pressure correction methods (see above [40,13,32,34,35,38,43,55,53,62,63,72,73]) or AUSM schemes (see [49,57]).

In this work, we are more interested in the second situation, in which sound waves play a weak role in the physical solution. Consequently, their precise description can be avoided. From the numerical point of view, this means that one would like to use large time steps compared to the small values of the Mach number. However, in order to do that, the scheme should be uniformly stable as well as consistent to the low Mach regimes.

One possible answer to this problem consists in adopting a fully implicit algorithm for the original compressible system. Such approach has been developed in [39] within the framework of multigrid methods. It allows large CFL numbers but it needs a preconditioning procedure to avoid the large number iterations needed for the resolution of the non linear system which originates from the implicit time discretization of the compressible equations [41,61,64] Recently, it has been shown that a fully implicit discretization is not the only strategy which permits to get all Mach number schemes. One alternative is represented by asymptotic preserving (AP) schemes [19,17,18,16,33,71,10,54,11,22]. They deal with different models which share the common characteristic of describing a multi-scale dynamic: i.e. a dynamic in which fast and slow scales coexist. These techniques allow computing the solution of such stiff problems while avoiding time step limitations directly related to the fast scale dynamic. This fast scale, in the context of this work, appears in the low Mach number regime when the pressure waves become fast compared to the rest of the dynamic. In addition, these AP methods lead to consistent approximations of the limit model (here the incompressible model) when the parameter which describes the fast scale dynamics goes to zero (here the Mach number). We stress that, even if the proposed methods in this work are specifically designed to avoid the fast scale resolution (remaining uniformly stable), if they are used with small time steps like those necessary for an explicit method, they are able to describe this fast dynamic with high accuracy. Thus, this approach also competitive compared to other methods designed to describe the fast pressure waves.

For the sake of completeness, let us briefly recall the general principle of such schemes. We start from one model of partial differential equations, $\mathcal{M}_\varepsilon$, depending on an arbitrarily small parameter ε and we assume that there exists a limit model, $\mathcal{M}_0$, which describes the dynamic when ε tends to 0. Their respective exact solutions are called $W_\varepsilon(x,t)$ and $W_0(x,t)$ for all space and time position (x,t), see Fig. 1 on the left.

Classical pairs of such related models are the inviscid Euler equations or the viscous Navier–Stokes model and their low Mach number limits. However, many other examples can be found in the literature, such as the Boltzmann equation and its hydrodynamic limit [23,24], the shallow-water model and its limit for low Froude numbers, the Vlasov–Maxwell model in the quasi-neutral limit, hyperbolic systems with stiff source terms, kinetic-fluid models in plasma physics or biology and many others. Here, the square of the Mach number plays the role of the small parameter ε, and when ε tends to zero, compressible flow equations converge to incompressible ones.