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Metric-based anisotropic mesh adaptation for 3D acoustic boundary element methods

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ABSTRACT

This paper details the extension of a metric-based anisotropic mesh adaptation strategy to the boundary element method for problems of 3D acoustic wave propagation. Traditional mesh adaptation strategies for boundary element methods rely on Galerkin discretizations of the boundary integral equations, and the development of appropriate error indicators. They often require the solution of further integral equations. These methods utilize the error indicators to mark elements where the error is above a specified tolerance and then refine these elements. Such an approach cannot lead to anisotropic adaptation regardless of how these elements are refined, since the orientation and shape of current elements cannot be modified.

In contrast, the method proposed here is independent of the discretization technique (e.g., collocation, Galerkin). Furthermore, it completely remeshes at each refinement step, altering the shape, size, and orientation of each element according to an *optimal* metric based on a numerically recovered Hessian of the boundary solution. The resulting adaptation procedure is truly anisotropic and independent of the complexity of the geometry. We show via a variety of numerical examples that it recovers optimal convergence rates for domains with geometric singularities. In particular, a faster convergence rate is recovered for scattering problems with complex geometries.

1. Introduction

We consider the scattering of time-harmonic acoustic waves by three-dimensional obstacles embedded within an unbounded, homogeneous domain. Various numerical methods can be used to solve such a problem, however a natural and popular approach is to reformulate the problem as a boundary integral equation. The main advantage of such a reformulation is to restrict the computational domain to the boundary of the obstacle and to exactly fulfill the outgoing radiation condition. The numerical solution of boundary integral equations is known as the Boundary Element Method (BEM) (often called the Method of Moments in the electromagnetic community). Despite the reduction in dimension of the computational domain, the main drawback of the BEM is the fully-populated nature of the system matrix. The cost of BEM simulations is thus prohibitively high when large-scale problems are concerned. Hence there has been a great deal of work since the inception of the BEM to reduce its computational cost.

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If no acceleration technique is used, the storage of such a system is $\mathcal{O}(N^2)$, where N is the number of degrees of freedom on the scatterer boundary (e.g., the number of nodes in the mesh of the discretized boundary). The cost of solving the dense system using a direct method such as Gaussian elimination requires $\mathcal{O}(N^3)$ flops, whereas solution via an iterative method such as GMRES requires $O(N_{\text{iter}}N^2)$, where N_{iter} is the number of iterations. In the last decades, different approaches have been proposed to speed up the solution of dense systems. The best-known method is probably the fast multipole method (FMM) originally proposed by Greengard and Rokhlin [35] which enables a fast evaluation of the matrix–vector product required by the iterative solver. Initially developed for N-body simulations, the FMM has since been extended to oscillatory kernels [25,34]. Now it is widely used in many application fields and has shown its capabilities in the context of mechanical engineering problems solved with the BEM [16,45]. An alternative approach designed for dense systems is based on the concept of hierarchical matrices ($\mathcal{H}$-matrices) [9]. The principle of $\mathcal{H}$-matrices is to partition the initial dense linear system, and then reduce it to a data-sparse one by finding sub-blocks in the matrix that can be accurately approximated by low-rank matrices. The efficiency of hierarchical matrices relies on the possibility to approximate, under certain conditions, the underlying kernel function by low-rank matrices. The approach has been shown to be very efficient for asymptotically smooth kernels (e.g., the Laplace kernel) and efficient in a pre-asymptotic regime for oscillatory kernels such as Helmholtz or elastodynamic kernels [19].

Mesh adaptation is an additional technique to reduce the computational cost of a numerical method. The principle is to optimize (or at least improve) the positioning of a given number of degrees of freedom on the geometry of the obstacle, in order to yield simulations with superior accuracy compared to those obtained via the use of uniform meshes. Adaptation is particularly important for scattering obstacles that contain geometric singularities, i.e., corners and ridges, which lead to a rapid variation of the surface solution near these singularities. For such problems, meshes graded toward these singularities must be employed in order to yield accurate approximations. In addition, for wave scattering problems, we may exploit the directionality of the waves in order to further reduce the number of degrees of freedom. The best strategy to achieve these goals is via so-called “anisotropic” mesh adaptation for which an extensive literature exists for volume-based methods such as the finite element method and the discontinuous Galerkin method [1]. However, there is relatively little research attention being paid to mesh adaptation in a boundary element context. One possible explanation is the large computational cost of standard BEMs. With the development of fast BEMs such as the Fast Multipole accelerated BEM (FM-BEM) [22] or $\mathcal{H}$-matrix accelerated BEM ($\mathcal{H}$-BEM) [12], the capabilities of the BEM are greatly improved such that efficient adaptive mesh strategies are needed not only to reduce further the computational cost, but also to certify the numerical results by assessing that the theoretical convergence order is observed during the computations.

In the BEM community, the majority of the research on mesh adaptation has been confined to isotropic techniques with a focus on the Laplace equation (see, e.g., the exhaustive review [27]) and extensions to the Helmholtz equation being made only fairly recently [5–7]. These isotropic techniques are usually based on *a posteriori* error analysis from which error indicators are derived. An indicator is then used to steer the mesh refinement by systematically marking and refining only elements where the error is above a specified threshold – a process known as Dörfler marking [15]. The derivation of appropriate local error estimators [26] is a significant challenge owing to the non-locality of boundary integral operators. This difficulty is the main reason why adaptivity for the BEM is a much less well-explored research topic in comparison to adaptivity for the FEM where the relevant operators are local differential operators. In many works convergence rates for error estimates are proven rigorously, e.g., [28,14,30]. However, it is seen that Dörfler refinement techniques do not usually recover the optimal convergence rates for 3D problems with anisotropic features [4]. Anisotropic variants of this strategy have been considered in [4,28] however with rectangular elements for cube or cube-like shapes (where all the ridges are right angles). For these shapes they obtain the optimal convergence rate, however for general shapes (or complex geometries), their approach would not perform as well. The additional drawback of previously published works is the problem-dependent or integral equation-dependent nature of the error estimates. Also, the error analysis of these methods requires a Galerkin discretization and hence a higher computational cost than, say, a collocation discretization.

The first novelty of the present work is the extension of metric-based anisotropic mesh adaptation (AMA) to the BEM. The metric-based AMA proposed in [38,39] does not employ a Dörfler marking strategy but rather generates a sequence of non-nested meshes with a specified complexity (proportional to the number of vertices or elements). The different meshes are defined according to a metric field derived from the evaluation of the linear interpolation error of the (unknown) exact solution on the current mesh. From a theoretical point of view, a continuous metric is derived from the Hessian of the exact solution. From a practical point of view, an approximate metric is derived from the numerical solution only (obtained via the BEM on a mesh). This approximate Hessian is based on the extension of typical (volumetric) derivative recovery operators [46] to the case of numerical boundary solutions. In AMA, the size, shape, and orientation of elements are adjusted simultaneously. The advantages of this approach are that it is ideally suited to solutions with anisotropic features, it is independent of the underlying PDE and discretization technique (collocation, Galerkin, etc.), and it is inexpensive. The metric-based AMA approach, as outlined above, has never been applied to the BEM. The purpose of this paper is to detail and report on the first application of metric-based AMA within a boundary element setting. Furthermore, we address some issues encountered when using an iterative solver for the FM-BEM on the resulting refined anisotropic meshes. In particular, we present two simple techniques to reduce the number of GMRES iterations required to achieve convergence when anisotropic elements are contained in the mesh.

The second novelty of this work is the combination of two acceleration techniques, namely metric-based anisotropic mesh adaptivity (AMA) and Fast Multipole acceleration. If no fast BEM is used, the capabilities of anisotropic mesh tech-