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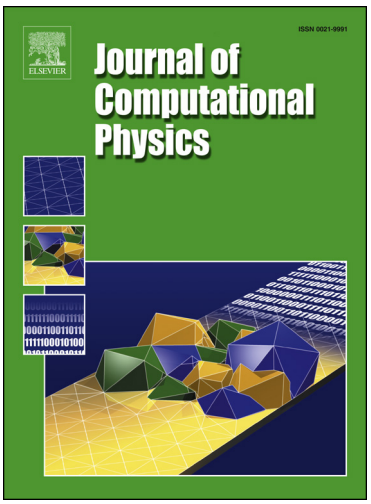

A hybrid approach to solve the high-frequency Helmholtz equation with source singularity in smooth heterogeneous media

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A hybrid approach to solve the high-frequency Helmholtz equation with source singularity in smooth heterogeneous media

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Abstract

We propose a hybrid approach to solve the high-frequency Helmholtz equation with point source terms in smooth heterogeneous media. The method is based on the ray-based finite element method (ray-FEM) [29], whose original version can not handle the singularity close to point sources accurately. This pitfall is addressed by combining the ray-FEM, which is used to compute the smooth far-field of the solution accurately, with a high-order asymptotic expansion close to the point source, which is used to properly capture the singularity of the solution in the near-field. The method requires a fixed number of grid points per wavelength to accurately represent the wave field with an asymptotic convergence rate of $\mathcal{O}(\omega^{-1/2})$, where ω is the frequency parameter in the Helmholtz equation. In addition, a fast sweeping-type preconditioner is used to solve the resulting linear system.

We present numerical examples in 2D to show both accuracy and efficiency of our method as the frequency increases. In particular, we provide numerical evidence of the convergence rate, and we show empirically that the overall complexity is $\mathcal{O}(\omega^2)$ up to a poly-logarithmic factor.

Keywords: Helmholtz equation, Babich's expansion, ray-FEM, NMLA