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Efficient methods with higher order interpolation and MOOD strategy for compressible turbulence simulations ☆

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A B S T R A C T

Efficient higher order interpolation schemes based on a multi-dimensional optimal order detection (MOOD) paradigm are developed coupled with the implicit time discretization scheme and further investigated for implicit large eddy simulation of compressible turbulence. The developed methodology utilizes higher-order either the upwind or the central interpolation to minimize numerical dissipation and meantime hybridizes shock capturing scheme that is also provided with higher-order interpolation to stabilize the solution. Simple and effective technique is proposed to apply the current method to an unsteady dual-time stepping scheme, which is to the best of the authors' knowledge the first time to develop the strategy for the MOOD application within an unsteady implicit time discretization framework. The resulting schemes are implemented in the simplified finite volume method that is constructed on non-uniform, curvilinear, multiblock structured grids. Numerical results for a comprehensive suite of both benchmark and practical problems demonstrate that the designed schemes simultaneously obtain the well-resolved broadband turbulence and the sharp shock profiles with considerable reduction in the computation cost.

1. Introduction

The main computational challenge of predicting compressible turbulence in general, and interactions between shock waves and turbulent flows in particular, arises from the contradictory properties of numerical methods designed to treat shocks and turbulence [1]. It is desirable that the numerical methods have both the properties of capturing discontinuities such as the shock waves with monotone profiles and high order low dissipation in smooth regions. Nevertheless, these two properties are often conflicting. It is well known that interpolations across discontinuities tend to generate spurious oscillations that can ultimately lead to a failure of the computation. And there has been an abundance of work to deal with the conflict between keeping the high accuracy of the solutions and stabilizing the computation. Among the broad range of algorithms in the literature, shock-capturing methods may be the most popular techniques [2,3].

For a long time the shock-capturing methods in computational fluid dynamics have been developed based on two quite different standpoints. One standpoint considers that the discrete zone consists of discontinuous solutions and the physics of wave propagation is accounted for by solving the Riemann problems at element interfaces. And these methods can be classified as the upwind schemes [3]. On the contrary, the other standpoint considers that the discrete zone consists

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of smooth solutions and the discontinuity is treated as continuous solutions with large gradient. The methods developed upon this standpoint can be classified as central schemes and the most typical techniques of such approaches are artificial viscosity methods [4,5]. Both methods have been widely applied within the framework of finite difference (FD) and finite volume (FV) discretization, and even more recently have been extended to finite element discretization and used as limiting strategies to stabilize solutions around discontinuities. While shock capturing methods of various degrees of sophistication exist in the literature, for direct numerical simulation (DNS) or large eddy simulation (LES) of the compressible turbulence with strong shocks, maybe now the most popular choices are still the FD discretization equipped with high order shock capturing schemes. Ref. [1] systematically investigates a comprehensive range of such high-order or high-resolution shock capturing schemes, the work of which is inspiring because it also provides suite of relevant test problems that are considered to best evaluate the strengths and weaknesses of the current algorithms. Also in [1] the so-called hybrid schemes [6–9] have been proved to be suitable methods for high-fidelity compressible turbulence simulations. More recently, a new concept of multi-dimensional optimal order detection (MOOD) approach was originally proposed in [10] and further extended in the following studies [11–13]. The MOOD is known as an a posteriori detection approach that can be exploited to design numerical schemes with favorable mathematic properties such as discrete maximum principle and positivity-preserving. Based on the MOOD concept, arbitrary hybrid schemes can be constructed without using any priori shock indicator, and this can be advantageous since it is concluded in [1] that the comprehensive nature of the test problems proved to be a challenge in terms of defining a shock sensor in the hybrid methods. However to the best of the authors' knowledge, currently the MOOD concept has been mainly investigated for non-broadband shock problems such as hyperbolic conservation laws, therefore it would be interesting to investigate this novel concept for more complex flows involving interactions between shocks and turbulence where contradictory requirements of numerical methods are required as we mentioned above.

On the other hand, when the shock capturing methods of various degrees of sophistication are proposed in the literature, explicit time schemes are often used for their simplicity and high order of accuracy. Even when these methods are investigated for compressible turbulence simulations, the explicit schemes are still widely applied because most work has focused the research on the spatial discretization. The explicit schemes are popular in the literature maybe also because most of the benchmark turbulent test cases are simplified and less computationally demanding when comparing to those practical turbulent problems like wall-bounded turbulent flows. It is well known that the convergence rate of explicit schemes slows down dramatically for wall-bounded turbulent flows and efficient implicit solution approaches should be employed for the small and stretched mesh elements that are required to resolve the thin boundary layers. Although in the context of the explicit time discretization adequate shock capturing schemes prove to be mathematically desired, whether these schemes are able to retain those carefully-designed properties when extended to implicit time discretization therefore still remain to be further investigated.

Based on the observation of current shock capturing schemes in particular the MOOD method, this work has developed efficient solution methods that explore the advantage of higher order interpolation and MOOD strategy in a framework of unsteady implicit time discretization. To be specific, we are in an attempt to achieve high efficient and accurate FV schemes that are built on higher-order interpolation and MOOD paradigm for implicit LES (ILES) application. Previous studies on the ILES [14–16] (and references therein) have shown that the built-in numerical dissipation of high-order/high-resolution schemes acts as a subgrid scale model accounting for the effects of the unresolved (turbulent) flow scales. Therefore the objective of the present study is to provide a suite of numerical methods that can be efficiently used in ILES method and especially can be used in the simulation of complex flows where shocks and turbulence are both present and interact dynamically. The key aspect of the work lies in efficient implementation of hybrid higher-order interpolation coupled with implicit MOOD strategy. We have constructed our methods on non-uniform, curvilinear, multiblock structured grids and have focused on not only the accuracy but also the efficiency of the designing scheme. The rest of the paper is organized as follows. Section 2 briefly introduces the governing equation and the FV formulation applied in the work. Section 3 describes the details of the high order interpolation schemes with MOOD strategy as well as the methods' implicit application. Numerical tests are carried out in Section 4 and some conclusions are drawn in Section 5.

2. The governing equation and the FV discretization

2.1. Governing equations

The compressible Navier–Stokes (NS) equations, excluding body forces and external heat sources, in curvilinear coordinates system are

$$\frac{\partial \boldsymbol{Q}}{\partial t}+\frac{\partial \boldsymbol{F}}{\partial \xi}+\frac{\partial \boldsymbol{G}}{\partial \eta}+\frac{\partial \boldsymbol{H}}{\partial \zeta}=\frac{\partial \boldsymbol{F}_v}{\partial \xi}+\frac{\partial \boldsymbol{G}_v}{\partial \eta}+\frac{\partial \boldsymbol{H}_v}{\partial \zeta} \tag{1}$$

where (ξ, η, ζ) is a system of curvilinear coordinates, $\boldsymbol{Q}$ are conservative variables, $\boldsymbol{F}$, $\boldsymbol{G}$, $\boldsymbol{H}$ and $\boldsymbol{F}_v$, $\boldsymbol{G}_v$, $\boldsymbol{H}_v$ are inviscid and viscous flux vectors in each curvilinear coordinate, respectively. The compressible Euler equation can be obtained by omitting the viscous flux in Eq. (1). By using the free stream density, sound speed, temperature and molecular viscosity, the non-dimensional NS equations remain the same form as Eq. (1) with the non-dimensional conservative, inviscid and viscous vectors now expressed as