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Reduced modeling of porous media convection in a minimal flow unit at large Rayleigh number

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ABSTRACT

Direct numerical simulations (DNS) indicate that at large values of the Rayleigh number (Ra) convection in porous media self-organizes into narrowly-spaced columnar flows, with more complex spatiotemporal features being confined to boundary layers near the top and bottom walls. In this investigation of high-Ra porous media convection in a minimal flow unit, two reduced modeling strategies are proposed that exploit these specific flow characteristics. Both approaches utilize the idea of decomposition since the flow exhibits different dynamics in different regions of the domain: small-scale cellular motions generally are localized within the thermal and vorticity boundary layers near the upper and lower walls, while in the interior, the flow exhibits persistent large-scale structures and only a few low (horizontal) wavenumber Fourier modes are active. Accordingly, in the first strategy, the domain is decomposed into two near-wall regions and one interior region. Our results confirm that suppressing the interior high-wavenumber modes has negligible impact on the essential structural features and transport properties of the flow. In the second strategy, a hybrid reduced model is constructed by using Galerkin projection onto a fully *a priori* eigenbasis drawn from energy stability and upper bound theory, thereby extending the model reduction strategy developed by Chini et al. (2011) [45] to large Ra. The results indicate that the near-wall upper-bound eigenmodes can economically represent the small-scale rolls within the exquisitely-thin thermal boundary layers. Relative to DNS, the hybrid algorithm enables over an order-of-magnitude increase in computational efficiency with only a modest loss of accuracy.

1. Introduction

Rayleigh–Bénard convection in a fluid-saturated porous layer is a prime example of a spatiotemporal pattern-forming system that exhibits rich nonlinear dynamics despite its comparably simple mathematical formulation [1–16]. Recently, there has been renewed interest in this system owing to the potential impact of buoyancy-driven convective flows on geological carbon dioxide (CO_2) storage, which is one promising means of reducing CO_2 emissions into the atmosphere [17–31]. In a wide horizontal porous layer uniformly heated from below and cooled from above, the basic conduction state becomes

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unstable via a stationary bifurcation when the Rayleigh number $Ra > 4\pi^2$ [1,2], and convection sets in as steady O(1) aspect-ratio rolls. As Ra increases, the thermal boundary layers generated by the steady roll cells become unstable and the resulting flow exhibits a series of transitions between periodic and quasi-periodic roll motions [3–8]. In a two-dimensional (2D) domain, the large-scale convective rolls are completely broken down for $Ra > 1300$, culminating in spatiotemporally chaotic dynamics [9].

The direct numerical simulations (DNS) performed by [11,14] reveal that, at sufficiently large Ra, thermal convection in porous media exhibits a three-layer wall-normal asymptotic structure: exquisitely thin diffusive boundary layers form adjacent to the upper and lower walls; the interior region is dominated by a nearly vertical columnar exchange flow ('mega-plumes') spanning the height of the domain; and between these regions, small proto-plumes grow from the boundaries and coalesce to drive the interior mega-plumes. As the Rayleigh number is increased, the time-mean mega-plume spacing L_m shrinks as a power-law scaling of Ra; e.g. $L_m \sim Ra^{-0.4}$ has been proposed by [11]. Moreover, the studies described in [32–34] indicate that at large Ra this mean inter-plume spacing approaches the minimal flow unit, above which the Nusselt number Nu becomes independent of the domain aspect ratio L. Collectively, these investigations suggest that the basic physics of high-Ra porous media convection can be investigated in a narrow domain in which the flow retains the three-region columnar structure but includes only a single pair of rising and descending mega-plumes, and heat is transported at the same rate as in wider domains.

The complexity of turbulent flows, even in ostensibly simple configurations like high-Ra porous media convection, generally necessitates the retention of hundreds of thousands or millions of degrees of freedom to ensure adequate resolution of all spatiotemporal scales of motion. Hence it is desirable to construct models with a reduced number of degrees of freedom that capture the essential nonlinear interactions over different spatial and temporal scales. As suggested above, one simple approach to reducing the number of degrees of freedom in turbulent flows is to study the dynamics in a small domain in which the turbulence nevertheless can be sustained. Of course, the use of small domains precludes long-wavelength interactions, but in many cases the fundamental features of the turbulence are still retained. In particular, we confirm in the following sections that high-Ra porous media convection in a minimal flow unit exhibits the same three-region columnar structure and heat flux as is manifest in wide domains. Therefore, in this study, we use the minimal flow unit to explore strategies for achieving further reductions in the number of degrees of freedom in simulations of porous media convection at large Ra.

One popular technique for constructing low-dimensional models involves the application of the Proper Orthogonal Decomposition (POD). In this method, an eigenfunction basis, whose modes can be ordered in terms of decreasing average energy content, is obtained by post-processing either experimental or numerical data. Galerkin projection of the governing partial differential equations (PDEs) onto this POD basis produces a system of ordinary differential equations (ODEs), and truncations of the resulting infinite set of ODEs yield low-dimensional models. Although POD has been used for model reduction for various turbulent flows including buoyancy-driven convection [35–44], a fundamental limitation of this approach is that extensive data sets are required from experiments or DNS *before* the reduced models can be constructed and their dynamics investigated. Moreover, although a small number of POD modes may capture most of the 'energy' of the infinite-dimensional dynamics, dynamically important modes having low average energy content may be omitted in the usual ordering employed in the construction of POD models [45].

In [45], a new, completely *a priori* low-dimensional modeling strategy was proposed. Specifically, eigenfunctions drawn from energy stability and upper bound theory were utilized to construct low-dimensional models of low-Ra porous media convection. Unlike widely-employed Fourier and Chebyshev (i.e. *a priori*) basis functions, the upper bound eigenbasis is extracted directly from the governing equations and is thereby naturally adapted to the dynamics at the given parameter values. For instance, as demonstrated in section 3, at large Ra certain upper-bound eigenfunctions exhibit boundary-layer structure, which is advantageous given that porous media convection self-organizes into narrow columnar plumes with more complex spatiotemporal features confined near the heated and cooled walls. In addition, it has been shown in [14] that the interior flow is a composite of a few low-wavenumber Fourier modes but is dominated by a single mode; the (wall-normal varying) Fourier amplitudes of the high horizontal-wavenumber modes are strongly localized near the upper and lower walls, where they superpose to comprise the small rolls and proto-plumes within the thermal and vorticity boundary layers.

Inspired by this emergent spatial and spectral structure of the columnar flow at large Ra, two complementary strategies are proposed to reduce the degrees of freedom required in numerical simulations of porous media convection. First, a *domain decomposition method* [46] is employed: the domain is partitioned into different regions in which different resolutions are used in conjunction with a Fourier–Chebyshev pseudospectral numerical scheme; secondly, a *hybrid reduced model* is constructed: at low horizontal wavenumbers, PDEs are solved using a standard Fourier–Chebyshev pseudospectral method (with domain decomposition), while at high wavenumbers, ODEs for the time-dependent coefficients of a small number of wall eigenmodes obtained from the upper-bound analysis are solved to economically capture the dynamics within the boundary layers.

The remainder of this paper is organized as follows. In the next section, we formulate the standard mathematical model of porous media convection. In section 3, the two complementary strategies for reducing the degrees of freedom in numerical simulations are described in detail. In section 4, computations employing these two approaches are performed at large Rayleigh number in the minimal flow unit, and the results are compared with those from resolved and under-resolved DNS. Finally, our conclusions are given in section 5.