Journal of Computational Physics 370 (2018) 1–24

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Localization of small obstacles from back-scattered data at limited incident angles with full-waveform inversion

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ARTICLE INFO

Article history:
Received 11 January 2018
Received in revised form 12 April 2018
Accepted 7 May 2018
Available online 29 May 2018

Keywords:
Inverse multiple-scattering
Locating small obstacles
Full-waveform inversion
Single-layer potential
Back-scattered data
Nonlinear gradient-based optimization

ABSTRACT

We investigate numerically the inverse problem of locating small circular obstacles in a homogeneous medium from multi-frequency back-scattered data limited to four angles of incidence. The main novelty of our paper is working with the position of the obstacles as parameter space in the frame work of full-waveform inversion (FWI) procedure. The computational cost of FWI is lowered by using a method based on single-layer potential. Reconstruction results are shown up to twenty-four obstacles, from initial guesses allowed to be far from the target. In experiments with six obstacles, we supplement the reconstruction with an analysis of the performance of the nonlinear conjugate gradient and quasi-Newton methods, in used with various line search algorithms.

1. Introduction

In this work, we use full-waveform inversion (FWI) to locate small circular obstacles in a homogeneous medium using multi-frequency backscattered data generated from a limited numbers of fixed angles. Obstacle localization problem has applications in materials imaging such as non-destructive testing using acoustic waves to detect defects, buried objects location, geophysical exploration and medical imaging, *cf.* [1,2]. For our localization problem, we work with impenetrable (hard or soft-scattering) non-overlapping obstacles. We assume that the number, size and type of the obstacles are known, and that the obstacles are located strictly inside a rectangular domain of interest in which experiments will be carried out to collect observed data. Motivated by physical experiments,[1] we also impose the following constraints in data collection for all testing frequencies (see illustration in Fig. 3),

- back-scattered data are obtained from the following angles: 0°, 90°, 180°, 270°;
- for each angle, the data are collected at a fixed number of 128 receivers (points).

In general, an inverse problem aims to reconstruct the unknown model/parameter which gives rise to an observed data $\mathbf{d}$. This is equivalent to solving for the inverse $\Phi^{-1}(\mathbf{d})$ of the forward map $\Phi : \mathcal{P} \to \mathcal{D}$, which assigns to a model in the parameter space $\mathcal{P}$ a corresponding data in the data space $\mathcal{D}$. In our approach, the parameter space $\mathcal{P}$ represents the set of ordered N_{par}-tuples of the coordinates of the N_{Obs} obstacles, *i.e.* $\mathcal{P} = \mathbb{R}^{N_{\text{par}}}$ with $N_{\text{par}} = 2N_{\text{Obs}}$. The inverse problem is solved

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[1] In our case, backscattered data arise when one single sensory device acts as both a source (by emitting almost planewaves) and receivers. Data of 128 points is the resolution set by the devices.

https://doi.org/10.1016/j.jcp.2018.05.011
0021-9991/

by minimizing the cost/misfit function $\mathcal{J} = \frac{1}{2}\|\Phi(\mathbf{p}) - \mathbf{d}\|^2$. Minimization is carried out by gradient-based optimization and corrects iteratively an initial guess in hope of converging towards the true model. In our work, the gradient of the cost function is calculated by the adjoint-state method, which avoids the computation of the Jacobian matrix of Φ.

In seismic inversion, the above methodology (nonlinear minimization of the cost function and adjoint-state method for its gradient) is called Full Waveform Inversion (FWI), see *e.g.* [3–6], and belongs to the so-called quantitative/iterative family, in contrast to qualitative/direct one. We refer to the introduction in [7] for a list of quantitative inversion references classified by their optimization method, as well as qualitative ones; for the latter family, see also [2,8]. Quantitative inversion has the advantage of being 'conceptually simple' and giving reconstruction with higher precision, *cf.* [9]; however, this family requires solving numerous direct and adjoint problems and can thus be computationally intensive. The general idea of using efficient methods (for the forward problem) in order to alleviate the overall computational cost in iterative inversion is not new, *cf.* [10,7] using fast solvers to solve forward obstacle and medium scattering. In our paper, we use an integral equation method based on single-layer potential (called FSSL) which was originally studied in [11,12], and was shown in our previous works [13,14] to be much more efficient, compared to Finite Element methods (FEM), in simulating diffraction by a large number of small impenetrable obstacles. In addition to this advantage, FSSL works more naturally with the current choice of parametrization (obstacle positions), *cf.* Remark 5.

The first main novelty of our results is in working directly with the position of the obstacles as parameters in the framework of nonlinear optimization with restrictive back-scattered data. In literature, most localization problems employ qualitative methods such as direct sampling, factorization, MUSIC, probe methods, *cf.* [15–20]. The remaining few which use quantitative methods are in fact (penetrable) medium reconstruction problem; they retrieve the location of obstacles from the profile of the reconstructed sound speed/contrast/conductivity function of the medium,[2] *cf.* [21,22]. For quantitative medium reconstruction without the final goal of localizing obstacles, we refer to *cf.* [23,24,7] and the references therein. The sensitivity and non-linearity of the scattered field and hence the forward map, with respect to the contrast function (in an inhomogeneous Helmholtz equation) are of different nature than that on the position of the obstacles (in a multiple-scattering problem with Helmholtz equation).[3] This distinction is even clearer, when scattering regions are impenetrable (as in our cases), for which multiple-obstacle scattering framework is more natural than inhomogeneous medium. Our work serves as an initial investigation of the feasibility of using the current choice of parameter space in FWI with line-search strategy, under the aforementioned restrictive data collection and starting from arbitrary initial guesses.[4]

Limited aperture data, especially back-scattered data, and limited number of radiation angles present great challenges, both theoretically and experimentally, since they increase the ill-posedness of the problem. Such restriction on data collection is a common feature in seismic inversion (that works with 'reflection data'), see [3,25], review [5] and the references therein. In general, ill-posedness is reduced by working with multi-frequency data, which is now a common technique; in seismic, *cf.* [26–29], in inverse scattering, see [30,31,24,10,7,32] for quantitative methods, and [19] and the references therein for qualitative ones. On the other hand, in the later context most of these references assume full-aperture data, with the exception of [33,19,34,8,15] in qualitative methods, and [35–38] in quantitative and the references therein. Furthermore, the number of angles of incidences and receiver points can be allowed to grow proportionally with the frequency, *cf.* [7]. In our experiments, these quantities are kept constant at all frequencies, which greatly affects the efficiency of higher frequencies, *cf.* Remark 8. Regardless of these constraints, we are able to retrieve up to twenty-four obstacles. These results are on the higher end (in terms of the number of obstacles) in both aforementioned references (for quantitative and qualitative methods).

Due to the lack of reference work implementing FWI with our parametrization, in choosing line search strategy [39], we have to investigate which choice of search direction and line search algorithm are most compatible with our problem. This motivates us to compare between nonlinear conjugated gradient (NLCG) and quasi-Newton, in combination with different line search algorithms, which is the second main novelty of our work. Quasi-Newton and NLCG were considered for medium or shape reconstruction in [40,41] and [22] respectively, however not in the framework of line search strategy and/or nonlinear optimization.

The remaining of the paper is organized as follows. Section 2 introduces FSSL and the discrete inverse problem. In Section 3, the derivative of the cost function and the frequency-hopping procedure are presented. Numerical experiments are in Section 4. Convergence comparison among different optimization methods for six obstacles is first carried out, after which the most reliable optimization method is tested with twelve and twenty-four obstacles.

[2] Obstacles are considered as compactly supported inhomogeneities in a homogeneous background. In inhomogeneous medium scattering, the obstacles are described by the contrast function q ($n = 1 - q$ is also called refractive index), and the direct problem is modeled as $(-\Delta - \kappa^2 n)\,u = 0$ (called inhomogeneous Helmholtz equation) coupled with an outgoing radiation condition at infinity. This can also be posed on bounded domain, *e.g.* in Electrical Impedance Tomography (EIT), in which the information of the obstacles is contained in the conductivity γ, and the direct problem modeled as $\nabla\cdot(\gamma\nabla u) = 0$ coupled with a Dirichlet condition on the boundary of the domain.

[3] For inhomogeneous Helmholtz equation, see Footnote 2. For multiple-scattering with Helmholtz equation, see the discussion in Section 2.

[4] As opposed to one generated by a direct imaging method as done in [23].