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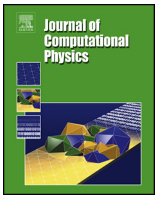

Efficient monolithic projection method for time-dependent conjugate heat transfer problems

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We propose herein an efficient monolithic projection method (MPM) to solve time-dependent conjugate heat transfer problems involving not only natural convection in the fluid domain and heat conduction in the solid domain, but also the thermal interaction between solid and fluid domains across the solid–fluid interface. We obtain a global discretized linearized system by advancing the buoyancy, nonlinear convection, and linear diffusion terms in time using the Crank–Nicolson scheme and introducing the second-order central finite difference in space along with linearizing the nonlinear convection terms in both momentum and energy equations. The energy equations are simultaneously and implicitly discretized in both solid and fluid domains with the implemented Taylor series expansion for thermal interaction normal to the interface without an involved sub-time step iteration. Approximated lower–upper decompositions and an approximate factorization are also imposed to speed up the computation. Thus, we obtain a non-iterative monolithic projection method over the entire domain. Numerical simulations of two-dimensional (2D) conjugate natural convection and 2D conjugate Rayleigh–Bénard convection and periodic forced flows are performed to investigate the numerical performance of the proposed method. Consequently, the MPM correctly predicts the solution of the conjugate natural convection problem involving strong thermal interactions and provides a more stable and efficient computation than the semi-implicit projection method proposed by Kim and Moin (1985) [21] with a loosely or strongly coupled algorithm for the solid–fluid interface, while preserving the second-order temporal and spatial accuracy. Finally, the proposed method reasonably simulates a typical real-world problem, namely conjugate heat transfer through double-pane windows, by considering 2D heat conduction in each pane of glass for three different climatic conditions. Using the proposed MPM, we also investigate the effects of the air layer thickness ranging from 5 mm to 40 mm on the averaged Nusselt number and the distribution of temperature as well as fluid motion.

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1. Introduction

Numerical studies of natural convection have been widely reported and are mainly based on the assumption that the bounding walls of enclosures are thermally thin [1–7]. Thus, the temperature inside the walls is assumed constant, which is reasonable for enclosures with high-conductivity walls and low-conductivity fluids. However, the thermal interaction between the fluids and solids must be considered in situations where the conductivities of the walls and fluids are comparable and/or the thickness of the walls is non-negligible. The thermal interaction between fluids and solids, which is also referred to as thermal conjugate heat transfer (CHT), is widely encountered in science and engineering applications, whereby heat conduction in solids is implicitly coupled with heat convection in adjacent viscous fluids through solid–fluid interfaces. CHT has attracted considerable attention over the last two decades because of its numerous engineering applications, such as thermal insulation design, solar equipment design, and energy-saving construction (e.g., use of double-pane windows), as well as heat transfer in the human body. The CHT problem involves not only the strong coupling between incompressible flows and heat transfers in the fluid domain but also the thermal coupling between the solid and the fluid across the solid–fluid interfaces, whereby heat transfers are convected and conducted by the background flow and solid, respectively.

Many researchers investigated these problems using various numerical and/or experimental methods to gain a deeper understanding of CHT coupling problems and thus improve design efficiency [8–16]. Kim and Viskanta [8] presented both numerical and experimental results for steady-state CHT in a 2D square air-filled cavity with four conductive walls, where the wall temperatures were measured using thermocouples, and the temperature distributions in the air-filled cavity were determined using a Mach–Zehnder interferometer. They found that the wall heat conduction reduces the average temperature differences across the cavity, partially stabilizes the flow, and decreases the natural convection heat transfer. Subsequently, Kaminski and Prakash [9] numerically analyzed the steady laminar natural convection flow in a square enclosure with one vertical heat-conducting wall of finite thickness and three walls of zero thickness using a control-volume-based finite difference method. They dealt with the nonlinearity, velocity–pressure coupling, and thermal interaction between the solid and the fluid by applying the iterative revised semi-implicit method for pressure-linked equations (SIMPLER) algorithm. Their method [9] simultaneously solves for the flow in the enclosure and the conduction in the wall, where a large number of iterations are required for very-high-Grashof-number cases, by considering three separate models for the solid conduction (i.e., 2D model, one-dimensional (1D) model, and a lumped parameter approach). They found that the temperature distribution in the thick wall shows significant 2D effects when the Grashof number is greater than 10^5. Furthermore, the solid–fluid interface temperature is significantly non-uniform, which causes the pattern in the enclosure to be asymmetric [9]. Joubert and Quéré [10] developed a numerical scheme, in which thermal coupling between fluid and sidewalls is iteratively achieved within each time step, to study 2D laminar buoyancy-driven flow in a rectangular enclosure with conductive sidewalls. This scheme combines an accurate pseudo-spectral Chebyshev approximation in space with the Adams–Bashforth scheme for nonlinear convection terms and the implicit Crank–Nicolson scheme for buoyancy and linear diffusion terms in time. Aydın [13] proposed a numerical method to study CHT through a double-pane window using a 1D approach for conduction in each pane under different climatic conditions of Turkey. He applied a finite difference method to the vorticity-stream function formulation and energy equations using the alternating direction implicit method of Peaceman and Rachford [17] to solve for the vorticity transport and the energy equations and an iterative successive over-relaxation method to solve the stream function equation. Furthermore, he showed that the energy loss through the double-pane window can be considerably reduced by optimizing the thickness of the air layer. Oztop et al. [14] examined conjugate mixed convection and conduction in a lid-driven enclosure with a thick bottom wall and a top moving wall using a numerical method available in the commercial Fluent software. They adopted the iterative SIMPLER method to deal with the pressure–velocity coupling and the quadratic upstream interpolation for convective kinematics (QUICK) method for discretizing the nonlinear convection terms in both momentum and energy equations. Numerical simulations [14] showed that heat transfer can be decreased by increasing the solid–fluid thermal conductivity ratio, Richardson number, and wall thickness ratios. To solve the transient thermal coupling of solid and fluids, Kazemi et al. [15] presented a loosely coupled finite volume partitioned algorithm with the Crank–Nicolson scheme for time integration and Picard iterations in the flow solver, where the second-order temporal accuracy is preserved. However, based on the stability analysis, they found that the partitioned algorithm [15] is unstable for large Fourier numbers. Kazemi et al. [16] recently developed a strongly coupled finite volume monolithic approach to provide a more stable algorithm than that in [15]. In [16], the high-order L-stable explicit first-stage singly diagonally implicit Runge–Kutta schemes were used for the CHT problems with different strengths of thermal interactions to advance the solution in time within each separate fluid and solid subdomain. Dirichlet–Neumann interface conditions were considered at the solid–fluid interface along the horizontal direction while performing sub-iterations at each stage. They verified that for thermally coupled problems with low thermal effusivities, the sub-iterations rapidly converged, and the convergence rate decreased as the strength of the thermal interaction increased; thus, more robust methods would be required [16]. Although these implicit methods [9,10,12–16] provide better stability and accuracy than an explicit method, iterative procedures are required for solving the CHT problems to retain the stability and accuracy of the partitioned solutions. Therefore, monolithic approaches are in great demand for simultaneously solving CHT problems, especially for cases involving high-strength thermal interactions.

The objective of the present study is not only to simultaneously solve the CHT problems in the fluid domain, solid–fluid interface, and solid domain in an efficient way, but also to adopt an implicit treatment for the solid–fluid interface conditions. In the fluid domain, the present method is based on the fluid solver presented in our previous work [7], in