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The stabilization of high-order multistep schemes for the Laguerre one-way wave equation solver

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ABSTRACT

This paper considers spectral-finite difference methods of a high-order of accuracy for solving the one-way wave equation using the Laguerre integral transform with respect to time as the base. In order to provide a high spatial accuracy and stability, the Richardson method can be employed. However such an approach requires high computer costs, therefore we consider alternative algorithms based on the Adams multistep schemes. To reach the stability for the one-way equation, the stabilizing procedures using the spline interpolation were developed. This made it possible to efficiently implement a predictor–corrector type method thus decreasing computer costs. The stability and accuracy of the procedures proposed have been studied, based on the implementation of the migration algorithm within a problem of seismic prospecting.

1. Introduction

Mathematical models based on the one-way wave equation (OWWE) are often considered in problems of ocean acoustics [1–3], seismic prospecting [4–7], as well as for setting non-reflecting boundary conditions [8–10]

$$\frac{\partial \tilde{u}}{\partial z} = -\mathrm{i}\frac{\omega}{c}\sqrt{1-\left(\frac{ck_x}{\omega}\right)^2}\,\tilde{u}, \tag{1}$$

where $\mathrm{i}=\sqrt{-1}$, $\tilde{u}\equiv\tilde{u}(k_x,z,\omega)$ is a wave component at the angular frequency ω, k_x is the horizontal wave number, c is the wave velocity, the vertical direction z is the extrapolation direction, i.e., the direction of one-way propagation, and the positive axis z is directed downward, i.e., toward increasing depth. The square-root operator can be formally represented by the Padé expansion [11–14]

$$\sqrt{1-\left(\frac{ck_x}{\omega}\right)^2} \approx \left[1-\sum_{s=1}^{n}\frac{\beta_s c^2 k_x^2}{\omega^2-\gamma_s c^2 k_x^2}\right], \tag{2}$$

where the coefficients γ_s, β_s for the propagation angle should be optimized [12,15]. The velocity model is assumed to be homogeneous, although it yields satisfactory results also for inhomogeneous media. In the latter case this model correctly keeps kinematics of waves, but not their amplitudes.

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The fundamental problem of the downward continuation algorithms of wave fields is the instability. If the coefficients γ_s, β_s are real, then for the angles around $\pi/2$ the argument of the square root becomes less than zero, the left-hand side of approximation (2) being complex, while the right-hand side is still real, hence causing inconsistency in the approximation. This results in an improper propagation of the evanescent mode which should exponentially decay. For stabilizing the real Padé approximation there are a few approaches [16–18] that allow suppressing unstable components of a wave field. On the other hand, setting the coefficients γ_s, β_s to be complex [19–22], a better consistency of the right-hand and the left-hand sides of approximation (2) can be attained. From the physical viewpoint this means the introduction of artificial dissipation that restricts an increase in instability for evanescent waves. However the presence of strong gradients of the velocity function, the use of the Marchuk–Strang type splitting for decreasing computer costs [23–25] and the simulation of high-frequency wave fields, can bring about the numerical instability. This is explained by the fact that optimal values of the coefficients γ_s, β_s are selected based on the principle of frozen coefficients for a homogeneous medium, while calculations are carried out for inhomogeneous velocity models with difference approximations and different decompositions for differential equations.

In addition to the problem of stability, one of the central computational problems of finite difference methods for solving equations (1), (2) is the inversion of the ill-conditioned systems of linear algebraic equations (SLAEs). For the two-dimensional problems direct methods for solving the SLAEs are rather efficient, but for the three-dimensional problems one has to use iterative procedures [26–28], which, as a rule, have low convergence rate for indefinite non-Hermitian matrices [29]. Direct methods for solving the SLAEs can be used for three-dimensional geometry. However, practical calculations show [30,31] that the number of mesh nodes has to be very small to calculate in a reasonable time.

To overcome these difficulties, a new approach to solving the problem (1), (2) was proposed in [32]. The solution is sought for as a series in Laguerre functions [33,34], while to increase the accuracy of spatial approximation the Richardson extrapolation [35] and dispersion-relation-preserving (DRP) schemes [36] are used. The coefficients of the Laguerre series expansion are recurrence relations and, therefore, can be calculated by solving the SLAEs with the same real well-conditioned matrix and different right-hand sides. To solve these SLAEs, it may be efficient to use the parallel dichotomy algorithm [37–39] in the two-dimensional case, and any iterative algorithm for positive-definite matrices in the three-dimensional case. In contrast to the Laguerre transform, the Fourier transform leads to SLAEs comprised of ill-conditioned matrices with complex entries. Another advantage of the algorithm [32] is that by combining the Laguerre transform with the Richardson method one can obtain a stable difference model of high-order accuracy and, at the same time, limit the growth of unstable harmonics when using the real Padé approximation of the operator (2).

Higher efficiency of calculations can be attained if one, instead of solving difference problems for the elliptic high-order operators, solves a sequence of problems for a second order operator, thereby decreasing the condition numbers of the SLAEs. For this we will consider a new method of the predictor–corrector type [40] based on multistep Adams finite difference schemes. Another feature of the algorithm proposed in the present paper is that higher accuracy of calculations is provided by using schemes of high approximation order, not by the Richardson extrapolation procedure, which increases the approximation order from second to fourth. However, it requires solving the initial equations on an auxiliary mesh with a doubled number of nodes. Unfortunately, practical calculations with multistep Adams schemes have shown that not only explicit but also implicit schemes of high-order accuracy forming the basis of the predictor–corrector method are unstable. To make the calculations stable, a new stabilizing algorithm based on spline filtering is proposed. It makes it possible to suppress instability of both the real Padé approximation (2) and the multistep schemes. Thus, the algorithm being proposed for solving the OWWE equation is computationally more efficient and has an order of accuracy that is higher than that of the method in [32], and its software implementation is easier and, hence, more efficient.

2. The stability analysis for a model 1D one-way wave equation

The aspects of stability in constructing a numerical method for solving the 2D OWWE occupy a highly important place. To investigate the stability let us first consider a model problem for the 1D OWWE:

$$\partial_t v + c\partial_x v = 0, \quad t > 0,\ x \in \mathbb{R} \tag{3}$$

with the initial condition $v(x, 0) = \varphi(x)$, $(\varphi(0) = \varphi(1))$ and the periodic boundary condition $v(0, t) = v(1, t)$.

To solve problem (3), let us consider the direct and inverse Laguerre transforms [34] of a function $g(t) \in L_2(0, \infty)$

$$\mathbb{L}\{g(t)\} = \bar{g}_m = \int_0^\infty g(t) l_m(\eta t) dt, \qquad g(t) = \mathbb{L}^{-1}\{\bar{g}_m\} = \sum_{m=0}^{\infty} \bar{g}_m l_m(\eta t), \tag{4}$$

where $l_m(\eta t) \equiv \sqrt{\eta}\exp(-\eta t/2) L_m(\eta t)$ are the orthogonal Laguerre functions, $L_m(t)$ is the Laguerre polynomial of m degree and $\eta > 0$ is the transformation parameter. Setting $\lim_{t\to\infty} g(t) = 0$, the following relations are valid [34,41]

$$\mathbb{L}\left\{\frac{d}{dt} g(t)\right\} = \frac{\eta}{2}\bar{g}_m + \Phi_1(\bar{g}_m), \tag{5}$$

where