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An iterative interface reconstruction method for PLIC in general convex grids as part of a Coupled Level Set Volume of Fluid solver

M. Skarysz *, A. Garmory, M. Dianat

Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, LE11 3TU, UK

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A B S T R A C T

Reconstructing the interface within a cell, based on volume fraction and normal direction, is a key part of multiphase flow solvers which make use of piecewise linear interface calculation (PLIC) such as the Coupled Level Set Volume of Fluid (CLSVOF) method. In this paper, we present an iterative method for interface reconstruction (IR) in general convex cells based on tetrahedral decomposition. By splitting the cell into tetrahedra prior to IR the volume of the truncated polyhedron can be calculated much more rapidly than using existing clipping and capping methods. In addition the root finding algorithm is designed to take advantage of the nature of the relationship between volume fraction and interface position by using a combination of Newton's and Muller's methods. In stand-alone tests of the IR algorithm on single cells with up to 20 vertices the proposed method was found to be 2 times faster than an implementation of an existing analytical method, while being easy to implement. It was also found to be 3.4–11.8 times faster than existing iterative methods using clipping and capping and combined with Brent's root finding method. Tests were then carried out of the IR method as part of a CLSVOF solver. For a sphere deformed by a prescribed velocity field the proposed method was found to be up to 33% faster than existing iterative methods. For simulations including the solution of the velocity field the maximum speed up was found to be approximately 52% for a case where 12% of cells lie on the interface. Analysis of the full simulation CPU time budget also indicates that while the proposed method has produced a considerable speed-up, further gains due to increasing the efficiency of the IR method are likely to be small as the IR step now represents only a small proportion of the run time.

1. Introduction

Among the most popular and widely used multiphase flow simulation methods are those based on a Volume of Fluid (VOF) formulation. In these methods, a function α is defined on a fixed grid which denotes the volume fraction of liquid within each cell. Piecewise linear interface calculation (PLIC) is used in geometric VOF methods to define the position of the interface within the cell. Besides geometric VOF [1,2], many high-order methods based on PLIC-VOF formulation have been implemented such as Moment of Fluid (MOF) [3], MOF coupled with Level Set [4] and Coupled Level Set and Volume

* Corresponding author.
E-mail address: m.skarysz@lboro.ac.uk (M. Skarysz).

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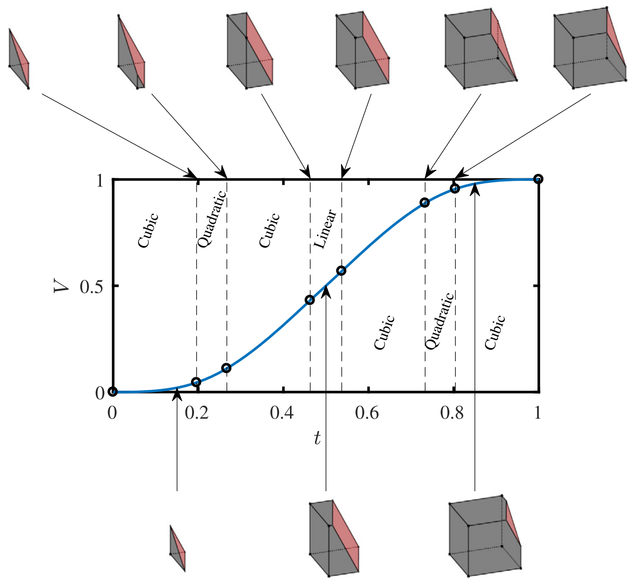

Fig. 1. Truncated cell volume as a function of plane position parameter t for a unit cube. Plane normal vector in spherical coordinates system $\vec{n} = [\phi = 65°, \theta = 70°]$.

of Fluid (CLSVOF) [5–8]. One common operation in each of these methods is an interface reconstruction (IR) step in which the interface between phases is defined within the cell. This is important for two reasons; the first is obtaining an exact position for the interface and the second is using this to define the volume flux through cell faces in the VOF advection step. This ensures mass conservation while limiting the smearing of the interface over multiple cells.

When using PLIC, IR is the process of finding a planar interface within a cell that satisfies a consistency condition, i.e., a plane of given normal direction truncates the polyhedral cell such that the volume on either side of the plane matches a known volume fraction. Any planar surface can be expressed as:

$$\vec{n} \cdot \vec{x} + D = 0 \tag{1}$$

where D is a constant and $\vec{n}$ is the normal vector of the plane which we assume is already given e.g., as a gradient of VOF or gradient of LS. The IR procedure is thus to determine D such that the plane splits the cell in two parts with given volume fraction α_T and $1 - \alpha_T$, where $\alpha_T \in [0, 1]$ and the subscript T indicates a target volume fraction. This involves solving the following equation for D:

$$\alpha(D) - \alpha_T = 0 \tag{2}$$

where $\alpha(D)$ is volume fraction obtained by splitting the cell by plane with constant D. We can re-scale the plane constant D to the parameter $t \in [0, 1]$such that $t = 0$ and $t = 1$ represent the cases of $\alpha_T = 0$ and $\alpha_T = 1$, respectively.

IR can refer to the entire multiphase interface in the computational domain or the local operation in the cell, which is also called local volume conservation enforcement. Here we will use *Interface Reconstruction* in both meanings, i.e., local and global. The complexity of the geometrical operations involved can result in significant CPU time cost. In this paper we consider the computational efficiency of the IR method, both in isolation and as part of full multiphase flow calculation. The latter is done to understand the potential for further improvements in a VOF type algorithm.

For IR problems in general 3D cells the most widely used and robust method was proposed by [9]. The solution is obtained in an iterative process by calculating the truncated volume for a series of values of the parameter D and comparing it to the prescribed target volume fraction until given tolerance is reached. An iterative root-finding method is needed such as Brent's [10] method. For simple geometries, more efficient methods were proposed by [11] for rectangular meshes and [12] for triangular and tetrahedral meshes, both based on an analytical approach. However as Eq. (2) changes its characteristics between linear, quadratic and cubic, as is shown for example in Fig. 1, any analytical approach must contain an initial step (usually iterative) which brackets the solution to the range where the degree of the equation is constant. Recently, analytical methods have been introduced for general convex cells [13–16] reducing CPU time cost considerably in comparison to previous iterative methods, for example in the case of a hexahedron cell the analytical method proposed by [14] is reported to be 4.7 times faster than Brent's method. The performance improvement rises to 5.9 for more complex cells.

In this paper, we propose an iterative method for interface reconstruction in a PLIC formulation that performs comparably with analytical methods and is simpler in implementation. The novelty of the paper is an efficient volume calculation