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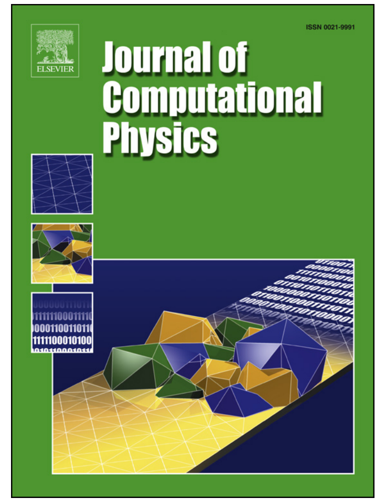

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Recovering an Unknown Source in a Fractional Diffusion Problem

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Abstract

A standard inverse problem is to determine a source which is supported in an unknown domain D from external boundary measurements. Here we consider the case of a time-independent situation where the source is equal to unity in an unknown subdomain D of a larger given domain Ω and the boundary of D has the star-like shape, i.e.

$$\partial D = \{q(\theta)(\cos\theta, \sin\theta)^\top : \theta \in [0, 2\pi]\}.$$

Overposed measurements consist of time traces of the solution or its flux values on a set of discrete points on the boundary $\partial\Omega$. The case of a parabolic equation was considered in [6]. In our situation we extend this to cover the *subdiffusion* case based on an anomalous diffusion model and leading to a fractional order differential operator. We will show a uniqueness result and examine a reconstruction algorithm. One of the main motives for this work is to examine the dependence of the reconstructions on the parameter α, the exponent of the fractional operator which controls the degree of anomalous behaviour of the process. Some previous inverse problems based on fractional diffusion models have shown considerable differences between classical Brownian diffusion and the anomalous case.

Keywords: fractional diffusion equation, inverse problem, uniqueness, unknown discontinuous source, Newton's method, Tikhonov regularization.

AMS subject classifications: 35R11, 35R30, 65M32.

1 Introduction

Our aim is to recover the location and shape of an extended source function $F = \chi_{_D}$ in a diffusion problem from making time-trace boundary measurements,

$$\begin{cases} {}^C D_t^\alpha u - \triangle u = \chi_{_D}, & (x,t) \in \Omega \times [0,T]; \\ u(x,0) = 0, & x \in \Omega; \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times [0,T]. \end{cases} \tag{1.1}$$

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