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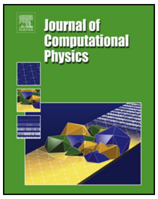

A plasma–vacuum interface tracking algorithm for magnetohydrodynamic simulations of coaxial plasma accelerators

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ABSTRACT

The resistive Magneto-hydrodynamic (MHD) model describes the behavior of a strongly ionized plasma in the presence of external electric and magnetic fields. For problems involving the plasma with an open boundary to very low pressure/vacuum regions, the continuum assumption is no longer valid in the entire domain of interest. For example, this is seen in a plasma accelerator where a dense plasma expands into a vacuum background. A common practice to deal with this issue is to assign small values of density and pressure to the vacuum regions and proceed to solve the continuum based MHD equations throughout the domain. We show that this approach fails to produce solutions consistent with the physics of the problem and can give rise to unacceptable artifacts such as spurious shocks. We develop a plasma–vacuum interface tracking approach to mitigate this problem. The plasma–vacuum interface is tracked in a physically consistent manner and the MHD equations are solved only in the regions that contain the plasma. The interface tracking is achieved using a face-flux formulation derived from the theoretical solution to a 1D free expansion problem. Coupled with a threshold based approach, the interface tracking is implemented for both explicit and implicit time stepping frameworks on generalized unstructured grids. Simulations of magnetized thermal plasma jets expanding into a vacuum background indicate plume profiles devoid of the unphysical shock obtained using the small background density approach. In the context of resistive MHD simulations, the interface tracking approach overcomes the numerical stiffness induced by specifying a large background resistivity in the vacuum regions, resulting in significant wall-clock time gains.

1. Introduction

The Magneto-hydrodynamics (MHD) governing equations are a continuum based description of electrically conducting gas flow (i.e. plasmas) subjected to external or induced electric and magnetic fields. The system of equations consists of the compressible Navier–Stokes equations that describe the flow behavior of the plasma coupled with the Maxwell's equations that describe the coupled electromagnetic fields. The electromagnetic fields affect the flow properties through the Lorentz force, which causes an acceleration of the plasma and Ohmic heating which increases the bulk plasma temperature. These electromagnetic fields are in turn generated by the plasma currents resulting in a tightly coupled non-linear equation

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system. The MHD equations model a wide variety of physical phenomena, e.g. in astrophysics [1], hypersonic aerodynamics [2], and in electric propulsion devices [3].

This paper is concerned with the development of a simulation approach for the study of MHD phenomena where a relatively dense plasma is sustained or accelerated into a vacuum or near-vacuum environment. An important application where such phenomena is encountered is the coaxial plasma accelerator. These devices utilize the Lorentz forcing caused by a self-induced magnetic field to accelerate the plasma to large velocities (∼10's km/s). The finite volume and discontinuous Galerkin formulations have been used successfully to solve the MHD equations, discretized on a domain representative of the accelerator geometry [3,4]. The MHD equations for the plasma accelerator problem present a high degree of mathematical stiffness owing to the disparate time scales associated with the different sub-physics such as flow convection and magnetic field diffusion. This has motivated the design of numerical schemes with time-stepping algorithms that overcome stiffness imposed by the operator with the smallest characteristic time scale. Sankaran et al. [5] used a flux splitting method with artificial diffusion and an explicit fractional time-stepping scheme to simulate a Magnetoplasmadynamic (MPD) thruster device. Their emphasis was the steady state simulation of the device, which motivated design of the fractional time stepping scheme. The time-stepping algorithm partially overcame restrictions imposed by the explicit integration of a stiff diffusion operator by modifying the number of times the convective flux was evaluated. Sitaraman and Raja [4] used a semi-implicit time stepping scheme for both the convective and diffusive fluxes to simulate the unsteady plasma acceleration process in two dimensions. The semi-implicit treatment of the fluxes was achieved by analytically deriving flux Jacobians and the resultant linear system was solved using a matrix free LU-SGS method [6]. The two-dimensional Multiblock Arbitrary Coordinate Hydromagnetic (MACH2) code [7,8] has been used to perform steady state simulations of the MPD thruster by explicitly resolving the fast plasma waves arising from the convective operator and implicitly treating the stiff diffusive terms [9]. Recently, Xisto et al. [10] developed an MHD extension of the PISO method [11] where an AUSM-MHD flux was used to perform steady state simulations of an MPD thruster.

In addition to the challenge of integrating operators with disparate time scales, the specific plasma accelerator operation mode that is the focus of this study poses special challenges. Here we are interested in the so-called 'deflagration mode', that involves the acceleration of a magnetized plasma into a hard vacuum background [12]. This is a highly unsteady phenomenon characterized by a smooth rarefaction of the dense plasma into vacuum. The increase in the Knudsen number (ratio of particle mean free path to the gradient length scale of the plasma) as one moves towards the rarefied regions of the plasma, leads to a breakdown of the continuum based model [13]. Consequently, an MHD numerical scheme that is derived from the continuum based formulation predicts a plasma jet profile that is inconsistent with the physics of free expansion [4]. Therefore, in addition to dealing with the disparate time scales, our application necessitates the design of an MHD numerical scheme that incorporates a physically consistent treatment of the plasma–vacuum expansion.

One approach is to use a hybrid continuum/particle solver where the rarefied regions are treated using a particle approach while the high density regions are modeled using a continuum formulation [14]. This method can be exceedingly expensive and the physics of interest in the plasma accelerator occurs at the continuum level [12] with an accurate resolution of the rarefied region being superfluous. An alternative approach that is commonly followed [4] is to replace the vacuum with a low density, low pressure gas followed by a continuum based numerical treatment of the entire domain. It is assumed that the expansion of the high density gas/plasma into the rarefied background would mimic free expansion into vacuum [4]. This approach has been consistently used [3–10] to perform steady state simulations of plasma accelerators. Here, the values of the background density/pressure are inconsequential towards the final solution since the transients associated with the plasma-background interaction eventually convect out of the simulation domain. However, in the context of unsteady simulations of plasma jets expanding into a vacuum background, the choice of a low background density/pressure as a substitute for vacuum results in unphysical plume profiles. Specifically, this approach leads to the formation of a strong shock at the region where the plasma jet interacts with the background gas. Such a behavior of the numerical solution is inconsistent with physics of free expansion, a process characterized by a single rarefaction wave. This issue was previously identified for a pure gas dynamic expansion into vacuum by Munz et al. [15], where they proposed a gas–vacuum interface tracking algorithm for the one-dimensional inviscid Euler's equations using an explicit time integration method. This approach is termed the Vacuum Riemann Solver (VRS). In this work, we extend the treatment of Munz to the 2D resistive MHD system in the context of a fully implicit time integration scheme. We present a threshold based plasma–vacuum interface tracking algorithm for the MHD simulation of coaxial plasma accelerators. The interface tracking scheme uses the underlying structure of the computational mesh to track the expanding plasma–vacuum interface in a physically consistent manner.

The remainder of this paper is organized as follows. Section 2 presents the resistive MHD model. Section 3.1 presents an overview of the spatial discretization schemes for the convective and magnetic diffusion operators followed by section 3.2 that outlines the fully-implicit time integration scheme. Sections 4.1 and 4.2 illustrate the failure of the low background density approach in capturing free expansion in a 2D plasma accelerator and a 1D gas-dynamic shock tube setup respectively. The shock formation is identified in section 4.2 as being associated with the convergence properties of the underlying convective flux scheme. The threshold based plasma–vacuum interface tracking algorithm is proposed in section 4.3 and is compared against a theoretical solution to a 1D free expansion problem. A 2D interface-tracking algorithm for generalized unstructured grids is presented in section 4.4 in the context of an explicit time stepping scheme for the Euler's equations. This approach is used to perform simulations of unmagnetized thermal plasma jets expanding into a vacuum background to illustrate the absence of a shock-front as compared to the low background density approach. Section 4.5 presents an