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Reduction of the discretization stencil of direct forcing immersed boundary methods on rectangular cells: The ghost node shifting method

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ABSTRACT

We present an analytical study of discretization stencils for the Poisson problem and the incompressible Navier–Stokes problem when used with some direct forcing immersed boundary methods. This study uses, but is not limited to, second-order discretization and Ghost-Cell Finite-Difference methods. We show that the stencil size increases with the aspect ratio of rectangular cells, which is undesirable as it breaks assumptions of some linear system solvers. To circumvent this drawback, a modification of the Ghost-Cell Finite-Difference methods is proposed to reduce the size of the discretization stencil to the one observed for square cells, i.e. with an aspect ratio equal to one. Numerical results validate this proposed method in terms of accuracy and convergence, for the Poisson problem and both Dirichlet and Neumann boundary conditions. An improvement on error levels is also observed. In addition, we show that the application of the chosen Ghost-Cell Finite-Difference methods to the Navier–Stokes problem, discretized by a pressure-correction method, requires an additional interpolation step. This extra step is implemented and validated through well known test cases of the Navier–Stokes equations.

1. Introduction

The immersed boundary method has been proven a successful treatment of complex boundaries in numerical simulations. The difficult generation of body-fitted grids is replaced by a modification of the governing equations near the boundaries, thus benefits of structured grids can be exploited. In addition, the immersed boundary method is particularly attractive when dealing with moving boundaries as it avoids re-meshing. Many different variants have been developed since the original immersed boundary method [1] as they explored coupling with different physical problems, different regimes, etc. Now the best approach to choose depends on physics constraints one wants to model. The review of Mittal and Iaccarino [2] explains this in details. The Direct Forcing approach, adopted in this article, first discretizes governing equations into a linear system; then applies boundary conditions near immersed boundaries to complete the linear system. This approach is similar to ordinary boundary conditions, but the non-Cartesian immersed boundaries leads to more complex interpolations. Both finite-difference discretization and finite-volume discretization can be used to apply immersed boundary conditions. This article focuses on specific aspects of the Ghost-Cell Finite-Differences approach, in the versions

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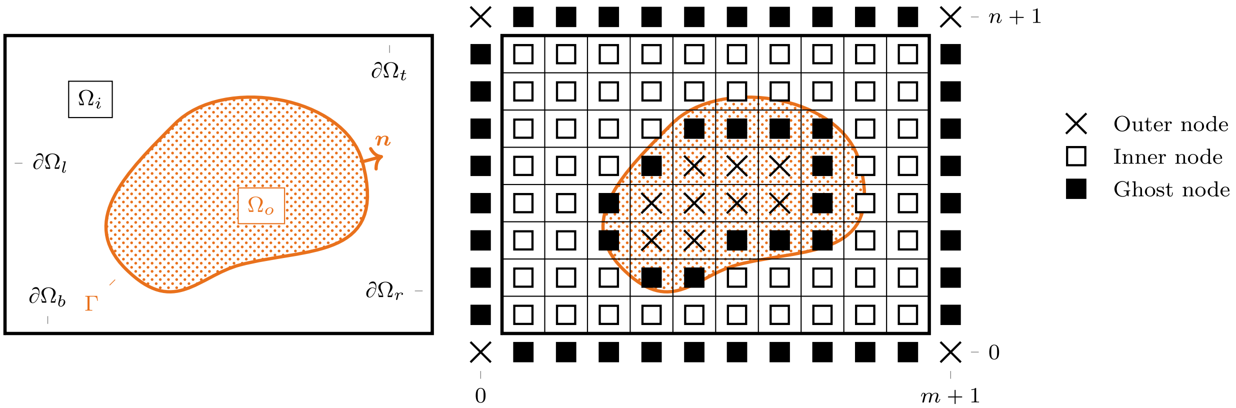

(a) Example of computational domain Ω split in an inner part Ω_i and an outer part Ω_o by the immersed boundary Γ.

(b) Example of Cartesian grid of resolution $m \times n$ with domain boundary extensions build upon Example Fig. 1a. Different symbols are used to distinguish inner, ghost, and outer cell nodes.

Fig. 1. Definitions and notations of the computational domain, the immersed boundary, the mesh, and node types.

proposed by Mittal et al. [3] and Coco and Russo [4], and applies them to the Poisson and Navier–Stokes problem, though this approach can be used to any boundary value problem.

Some linear system solvers constrain the matrix profile, i.e. cells allowed to be nonzero, to take advantage of properties that allow to achieve the highest performance in large parallel computations. For instance, geometric multi-grid algorithms SMG [5,6] and PFMG [7,8] of the *hypre* library can be used only with band matrices with 9/27 points stencils in 2D/3D for SMG and PFMG. More generic solvers and preconditioners such as the algebraic BoomerAMG of the *hypre* library are less efficient (see [9–11]) even if they are competitive. In addition, band matrices requires less memory than more generic sparse matrix format, such as Compressed Row Storage, because their topology do not need to be stored for each line of the matrix. The Ghost-Cell Finite-Differences methods proposed by Mittal et al. [3] and Coco and Russo [4] do not fit in band matrices generated by the discretization of the equations, whose stencil is broken for rectangular cells. These methods are based on the “closest point of the boundary” (as explained in Sec. 2.2) which does not take into account the properties of the numerical grid. This article proposes a modification of these methods to adapt them on band matrices in the case of rectangular cells. This article is organized as follows: the two Ghost-Cell Finite-Differences methods [3,4] are presented in Sect. 2 through the Poisson problem. Their impact on the discretization stencil is studied in Sect. 3, and the Ghost Node Shifting Method is defined in Sect. 4. Numerical simulations validate the proposed method in Sect. 5 for the Poisson problem with Dirichlet and Neumann boundary conditions. Finally, the Ghost-Cell Finite-Differences methods are introduced into the incompressible Navier–Stokes equations in Sect. 6 and some validations are presented in Sect. 7.

2. Immersed boundary methods with the Poisson problem

This section presents the Finite-Differences Ghost-Fluid immersed boundary methods applied to the Poisson problem, as found in the literature. For the sake of simplicity, methods are described in the 2-dimensional real space.

2.1. Problem description and discretization

The *computational domain* Ω is a rectangle of the 2-dimensional real space. It is split into two sub-domains by the *immersed boundary* Γ: one is the *inner part* Ω_i, while the other is the *outer part* Ω_o. The boundary Γ is oriented by a unit vector $\boldsymbol{n}$ from the outer domain to the inner domain. The *domain boundary* $\partial\Omega$ decomposes in four line segments $\partial\Omega_b$, $\partial\Omega_r$, $\partial\Omega_t$, $\partial\Omega_l$, where the letters b, r, t, l stand for bottom, right, top, left, respectively. These definitions are illustrated in Fig. 1a.

Poisson problem. We consider $u : \Omega_i \to \mathbb{R}$, a scalar field restricted to the inner domain, solution of the Poisson equation with a known *source field* $f : \Omega_i \to \mathbb{R}$:

$$\Delta u = f\,, \qquad \text{in } \Omega_i\,, \tag{1a}$$

$$u = D\,, \qquad \text{on } \partial\Omega_D \cup \Gamma_D\,, \tag{1b}$$

$$\partial_{\boldsymbol{n}} u = N\,, \qquad \text{on } \partial\Omega_N \cup \Gamma_N\,. \tag{1c}$$

Here boundaries have been split into a “Dirichlet” part $\partial\Omega_D \cup \Gamma_D$ and a “Neumann” part $\partial\Omega_N \cup \Gamma_N$; boundary conditions $D : \Gamma \to \mathbb{R}$ and $N : \Gamma \to \mathbb{R}$ are known.