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Multilayer shallow water models with locally variable number of layers and semi-implicit time discretization

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ABSTRACT

We propose an extension of the discretization approaches for multilayer shallow water models, aimed at making them more flexible and efficient for realistic applications to coastal flows. A novel discretization approach is proposed, in which the number of vertical layers and their distribution are allowed to change in different regions of the computational domain. Furthermore, semi-implicit schemes are employed for the time discretization, leading to a significant efficiency improvement for subcritical regimes. We show that, in the typical regimes in which the application of multilayer shallow water models is justified, the resulting discretization does not introduce any major spurious feature and allows again to reduce substantially the computational cost in areas with complex bathymetry. As an example of the potential of the proposed technique, an application to a sediment transport problem is presented, showing a remarkable improvement with respect to standard discretization approaches.

1. Introduction

Multilayer shallow water models have been first proposed in [2] to account for the vertical structure in the simulation of large scale geophysical flows. They have been later extended and applied in [5,4,6]. This multilayer model was applied in [3] to study movable beds by adding an Exner equation. A different formulation, to which we will refer in this paper, was proposed in [26], which has several peculiarities with respect to previous multilayer models. The model proposed in [26] is derived from the weak form of the full Navier–Stokes system, by assuming a discontinuous profile of velocity, and the solution is obtained as a particular weak solution of the full Navier–Stokes system. The vertical velocity is computed in a postprocessing step based on the incompressibility condition, but accounting also for the mass transfer terms between the internal layers. In [25], this multilayer approach is applied to dry granular flows, for which an accurate approximation of the vertical flow structure is essential to approximate the velocity-pressure dependent viscosity.

Multilayer shallow water models can be seen as an alternative to more standard approaches for vertical discretizations, such as natural height coordinates (also known as z-coordinates in the literature on numerical modelling of atmospheric and oceanic flows), employed e.g. in [11,16,19], terrain following coordinates (also known as σ-coordinates in the literature), see

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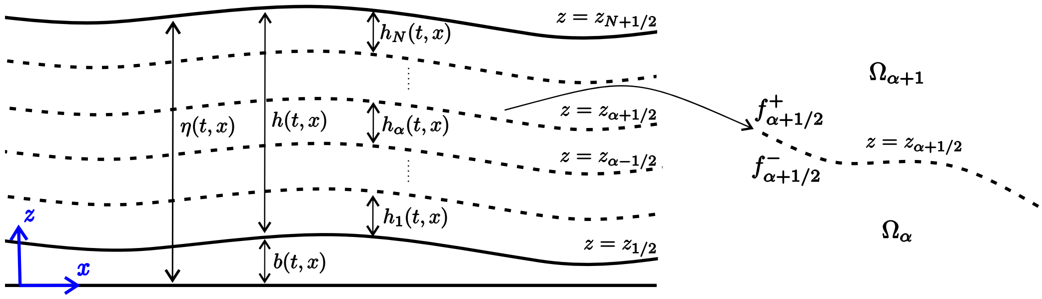

Fig. 1. Sketch of the domain and of its subdivision in a constant number of layers.

e.g. [31], and isopycnal coordinates, see e.g. [9,18]. Each technique has its own advantages and shortcomings, as highlighted in the discussions and reviews in [1,11,12,32]. Multilayer approaches are appealing, because they share some of the advantages of z-coordinates, such as the absence of metric terms in the model equations, while not requiring special treatment of the lower boundary. On the other hand, multilayer approaches share one of the main disadvantages of σ-coordinates, since they require, at least in the formulations employed so far, to use the same number of layers independently of the fluid depth. Furthermore, an implicit regularity assumption on the lower boundary is required, in order to avoid that too steeply inclined layers arise, which would contradict the fundamental hydrostatic assumption underlying the model.

In this work, we propose two strategies acting at the same time to make multilayer models more efficient and fully competitive with their z- and σ-coordinates counterparts. On one hand, we propose a novel discretization approach, in which the number of vertical layers can vary over the computational domain. We show that, in the typical regimes in which the application of multilayer shallow water models is justified, the resulting discretization does not introduce significant errors and allows to reduce substantially the computational cost in areas with complex bathymetry. In this way, multilayer approaches become fully competitive with z-coordinate discretizations for large scale, hydrostatic flows. Furthermore, efficient semi-implicit discretizations are applied for the first time to the discretization of the free surface gradients and the flow divergence in multilayer models. Notice that a semi-implicit approach for the discretization of vertical viscosity and friction terms has instead been introduced in [4,26]. In order to further simplify the presentation, we only introduce the discretization for an x–z vertical slice, even though both, the multilayer approach (see [26]) and any of the methods presented, can be generalized to the full three dimensional case. In this paper, again for simplicity, we have restricted our attention to constant density flows. An extension to variable density problems in the Boussinesq regime will be presented in a forthcoming paper. However, as a first step, we present in Appendix A a detailed description of the coupled discretization of a tracer equation. Not only it is the basis for the variable density extension, but, as shown in [30], the coupling of this equation to the discretized continuity equation is not a trivial issue and it is very important to verify compatibility conditions between the discrete continuity equation and the discrete tracer equations.

In section 2, the equations defining the multilayer shallow water models of interest will be reviewed. In section 3, the spatial discretization is introduced in a simplified framework, showing how the number of layers can be allowed to vary over the computational domain. In section 4, some semi-implicit time discretizations are introduced for the model with a variable number of layers. Results of a number of numerical experiments are reported in section 5, showing the significant efficiency gains that can be achieved by combination of these two techniques. Some conclusions and perspectives for future work are presented in section 6.

2. Multilayer shallow water models

We consider the multilayer shallow water model described pictorially in Fig. 1. In this approach, N subdivisions Ω_α, $\alpha = 1, \ldots, N$ of the domain Ω are introduced in the vertical direction. We denote by h_α the height of the layer α and by $h = \sum_{\alpha=1}^N h_\alpha$ the total height. Note that $\Omega = \bigcup_{\alpha=1}^N \Omega_\alpha$ and that each subdomain Ω_α is delimited by time dependent interfaces $\Gamma_{\alpha\pm\frac{1}{2}}(t)$, that are assumed to be represented by the one valued functions $z = z_{\alpha\pm\frac{1}{2}}(t,x)$. These interfaces can be written as $z_{\alpha+1/2} = z_{1/2} + \sum_{\beta=1}^{\alpha} h_\beta$, depending on the thicknesses h_α, where $z_{1/2} = b(x)$ is a function describing the bottom.

Given a function f we also define as in [26], for $\alpha = 0, 1, \ldots, N$,

$$f^-_{\alpha+\frac{1}{2}} := (f_{|\Omega_\alpha(t)})_{|\Gamma_{\alpha+\frac{1}{2}}(t)} \quad \text{and} \quad f^+_{\alpha+\frac{1}{2}} := (f_{|\Omega_{\alpha+1}(t)})_{|\Gamma_{\alpha+\frac{1}{2}}(t)}.$$

Obviously, if the function f is continuous,

$$f_{\alpha+\frac{1}{2}} := f_{|\Gamma_{\alpha+\frac{1}{2}}(t)} = f^+_{\alpha+\frac{1}{2}} = f^-_{\alpha+\frac{1}{2}}.$$

Note that this subdivision corresponds to the vertical discretization of the domain, which, a priori, is not related to the characteristics neither of the flow nor of the domain.