Journal of Computational Physics 364 (2018) 274–297

Contents lists available at ScienceDirect

Journal of Computational Physics

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Variance reduction method for particle transport equation in spherical geometry

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ARTICLE INFO

Article history:
Received 11 May 2017
Received in revised form 25 January 2018
Accepted 9 February 2018
Available online 15 March 2018

Keywords:
Transport equation
Monte Carlo
Radiative transfer
Inertial confinement fusion
Variance reduction

ABSTRACT

This article is devoted to the design of importance sampling method for the Monte Carlo simulation of a linear transport equation. This model is of great importance in the simulation of inertial confinement fusion experiments. Our method is restricted to a spherically symmetric idealized design: an outer sphere emitting radiation towards an inner sphere, which in practice should be thought of as the hohlraum and the fusion capsule, respectively. We compute the importance function as the solution of the corresponding stationary adjoint problem. Doing so, we have an important reduction of the variance (by a factor 50 to 100), with a moderate increase of computational cost (by a factor 2 to 8).

1. Introduction

In inertial confinement fusion (ICF) experiments, a small ball of hydrogen (the target) is submitted to intense radiation by laser beams. These laser beams are either pointed directly to the target (direct drive approach), or pointed to gold walls of a hohlraum in which the target is located (indirect drive approach, see Fig. 1). These gold walls heat up, emitting X-rays toward the target. The outer layers of the target are heated up, hence ablated. By momentum conservation, the inner part of the target implodes (this is usually called the rocket effect). Hence, the pressure and temperature of the hydrogen inside the target increase, hopefully reaching the thermodynamical conditions for nuclear fusion. This process is summarized in Fig. 2.

The numerical simulation of such an experiment involves many physical phenomena such as hydrodynamics, radiation transfer, neutronics, etc. In the present article, we focus on the simulation of radiation, that is, the transmission of the (X-ray) energy to the target. A simplified model for this is the grey radiative transfer equation:

$$\begin{cases} \partial_t u + \mathbf{\Omega} \cdot \nabla u + \kappa_t u = \kappa_s \displaystyle\int_{S^2} u(t, \mathbf{x}, \mathbf{\Omega}') k(\mathbf{x}, \mathbf{\Omega}', \mathbf{\Omega}) d\mathbf{\Omega}' + Q(\mathbf{x}) \\ u(t=0, \mathbf{x}, \mathbf{\Omega}) = g(\mathbf{x}, \mathbf{\Omega}), \end{cases} \tag{1.1}$$

where the solution u is the radiation intensity and depends on the time t, the position $\mathbf{x} \in \mathbb{R}^3$, the direction of propagation $\mathbf{\Omega} \in S^2$. The term $Q(\mathbf{x})$ represents a source of radiation. In the present case, $Q(\mathbf{x})$ is a modelling of the emission of X-rays

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https://doi.org/10.1016/j.jcp.2018.02.015
0021-9991/© 2018 Published by Elsevier Inc.

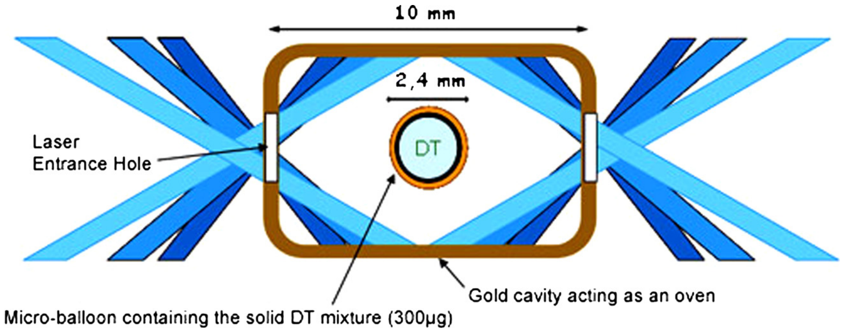

Fig. 1. Schematic view of the Hohlraum and the target.

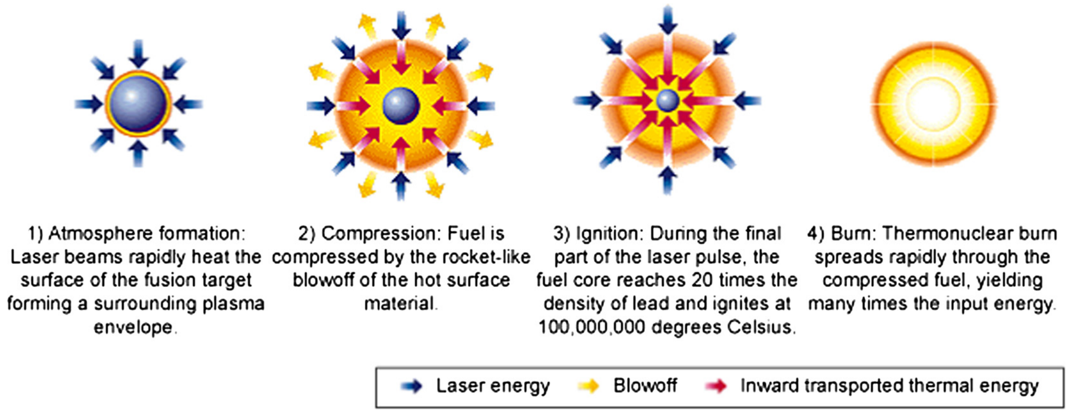

Fig. 2. The concept of ICF (inertial confinement fusion) taken from http://www.lanl.gov/projects/dense-plasma-theory/background/dense-laboratory-plasmas.php.

by the hohlraum walls. Furthermore, κ_t is the total cross-section. It satisfies $\kappa_t = \kappa_a + \kappa_s$, where $\kappa_a \geq 0$ is the absorption cross-section and $\kappa_s \geq 0$ the scattering cross-section. The kernel $k(\mathbf{x}, \mathbf{\Omega}', \mathbf{\Omega})$ is a probability density with respect to $\mathbf{\Omega}'$ and $\mathbf{\Omega}$, that is, $k \geq 0$ and $\displaystyle\int k(\mathbf{x}, \mathbf{\Omega}', \mathbf{\Omega}) d\mathbf{\Omega}' = \int k(\mathbf{x}, \mathbf{\Omega}', \mathbf{\Omega}) d\mathbf{\Omega} = 1$. Note that we have assumed here that we use units such that the speed of light is $c = 1$.

Equation (1.1) may be simulated using a Monte Carlo method. If so, the probability distribution k may be interpreted as the probability density associated to the new direction propagation $\mathbf{\Omega}$ for a particle having a shock with initial direction $\mathbf{\Omega}'$. In Monte Carlo simulations of such situations, variance reduction methods are important to reduce the statistical noise. Indeed, as the target implodes, hydrodynamic instabilities develop, which are a source of energy loss. Should this loss be too important, the experiment would be compromised. Thus, it is important to have a precise numerical description of these instabilities. In the case of Monte Carlo simulations, this implies a statistical noise as small as possible (at least smaller than the amplitude of the instabilities). A small variance is particularly important on the target boundary.

A widely used reduction variance technique in such a situation is the importance sampling method. It may be summarized as follows:

1. Calculate the importance function (in our case, the solution of the adjoint equation).
2. Use the importance function to modify the transport equation, and apply a Monte Carlo method.
3. Calculate the forward intensity from 1 and 2.

Importance sampling is a well-known reduction variance method, which has been applied to transport problems in many situations. We refer for instance to the textbooks [10], [14] for a general presentation. The key-point in such a method is the way one computes the importance function. If it is solution to the adjoint problem, then one achieves a zero-variance method. However, solving the adjoint problem is at least as difficult as solving the direct problem at hand. Therefore, many methods using approximations of the adjoint solution have been developed. This is the spirit of the exponential transform (see [6] and [10]). In some situations, a diffusion approximation is used for this calculation, as for instance in [17]. In other situations, discrete ordinates approximation is preferred [15]. The method which is the closest to the one presented here is probably [2], in which the adjoint equation is formulated as an integral equation, and solved using a space discretization. An importance difference is, however, that when solving the adjoint problem, the scattering is neglected in [2]. Here, we use