Journal of Computational Physics 364 (2018) 347–364

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Eigenmode computation of cavities with perturbed geometry using matrix perturbation methods applied on generalized eigenvalue problems

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A R T I C L E   I N F O

Article history:
Received 12 October 2017
Received in revised form 6 March 2018
Accepted 6 March 2018
Available online 12 March 2018

Keywords:
Generalized eigenvalue problem
Matrix perturbation
Modal sensitivity analysis
Slater's theorem
Eigenvalue reanalysis
Microwave cavity

A B S T R A C T

Generalized eigenvalue problems are standard problems in computational sciences. They may arise in electromagnetic fields from the discretization of the Helmholtz equation by for example the finite element method (FEM). Geometrical perturbations of the structure under concern lead to a new generalized eigenvalue problems with different system matrices. Geometrical perturbations may arise by manufacturing tolerances, harsh operating conditions or during shape optimization. Directly solving the eigenvalue problem for each perturbation is computationally costly. The perturbed eigenpairs can be approximated using eigenpair derivatives. Two common approaches for the calculation of eigenpair derivatives, namely modal superposition method and direct algebraic methods, are discussed in this paper. Based on the direct algebraic methods an iterative algorithm is developed for efficiently calculating the eigenvalues and eigenvectors of the perturbed geometry from the eigenvalues and eigenvectors of the unperturbed geometry.

1. Introduction

Generalized eigenvalue problems are of crucial relevance and a standard problem in computational sciences. Generalized eigenvalue problems arise e.g. from the discretization of the Helmholtz equation for electromagnetic fields by means of the finite element method. The eigenvectors of the generalized eigenvalue problem contain information on the field distributions of resonances of the structure under concern while the eigenvalues contain the frequencies of the resonances. These resonances are required to characterize the electromagnetic properties of the structure. The generalized eigenvalue problems are also widely encountered in mechanical systems [1–3] for instance in the modal analysis of mechanical structures such as the vibration of a cantilever beam. Geometrical perturbations of the structure lead to new eigenvalue problems with different system matrices. The geometrical perturbations may arise by the manufacturing tolerances during fabrication.

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https://doi.org/10.1016/j.jcp.2018.03.012

In addition, harsh operating conditions, e.g. cryogenic temperature for superconducting accelerators, can also deform the shape of the structure and thus shift the operating frequency. During the shape optimization in the design stage, the eigenvalue problem is formed and has to be solved for a large number of slightly different geometries. Solving the generalized eigenvalue problem for each case is computationally costly. Matrix perturbation methods are useful for efficiently calculating eigenvalues and eigenvectors of the perturbed or modified structure based on the eigenvalues and eigenvectors of the unperturbed structure.

The approximation of the perturbed eigenmodes can be treated numerically or analytically. Analytic perturbation methods in the context of electromagnetism are derived from the direct manipulation of Maxwell's equations. Such methods approximate the perturbed modes by means of a set of initial modes of the unperturbed geometry [4–6]. The disadvantage of these methods is that a large number of modes of the unperturbed structure is needed to accurately characterize one particular mode of the perturbed structure. Furthermore, these methods require the perturbed geometry to be enclosed within the unperturbed geometry. In this paper numerical perturbation methods applied on generalized eigenvalue problems are studied.

Matrix perturbation theory applied on eigenvalue problems, often in literature referred to as eigenvalue reanalysis, has received much attention in last decades [7–16]. Baldwin and Hutton [7] reviewed the developments in modification techniques such as Rayleighs method, sensitivity and perturbation approaches prior to 1985. A high accuracy step-by-step perturbation method applied on generalized eigenvalue problem is presented in [8]. In this method, a large modification to the system matrices is divided into small changes and the perturbation analysis is carried out step-by-step in an iterative manner. A similar step-by-step perturbation technique is used in [13] for electromagnetic problems. In [10], the Kirsch combined approximation (CA) method for static displacement reanalysis [17] is combined with Rayleigh quotient to reanalyze the eigenvalues of modified structures. In [11], Kirsch applied the CA method to eigenproblem reanalysis. In this method, a transformation matrix is formed and used to project the system matrices into a lower dimensional subspace. The modified eigenvector is approximated by a reduced set of basis vectors and the coefficients are found by solving a reduced eigenproblem formed by projected system matrices. A new method for eigensolution reanalysis based on Neumann series expansion and epsilon-algorithm is introduced in [12]. In [14], it is shown that sensitivity data such as first and second order eigenvector sensitivity data could be used as basis vectors for eigenmode reanalysis. An adaptive eigenvalue reanalysis method for a Genetic Algorithm to optimize mechanical structures is presented in [15].

The perturbed eigenpairs can be approximated by means of the eigenpair derivatives. Two common approaches for the calculation of the eigenpair derivatives are modal superposition methods and direct algebraic methods. The idea behind modal superposition method is similar to the analytic methods, i.e. expressing the eigenvector derivative as a superposition of the unperturbed eigenvectors [18]. In contrast, direct methods calculate the eigenvector derivatives by solving a linear system of equations so that only the eigenpair whose derivative is to be calculated is required. In 1976, Nelson [19] introduced a powerful yet simple algorithm for the calculation of eigenvector derivatives for distinct eigenvalues of the standard eigenvalue problem. Nelson's method, unlike previous methods which required computing all or most of the eigenvectors, needs only those eigendata that are to be differentiated. Nelson presented his approach for non-symmetric standard eigenvalue problems. The method preserves the band structure of the matrices. Dailey [20] presented a reformulation of the method for generalized eigenvalue problems with symmetric matrices. Nelson's method however works only for distinct eigenvalues. Ojalvo [21] presented a method for calculating eigenpair derivatives in case of repeated eigenvalues. The method was further completed by Mills-Curran [22] and Dailey [20]. In [23,24], the calculation of the eigenpair derivatives for special cases, where eigenvalues and eigenvalue derivatives are repeated, is presented. An algorithm for the calculation of eigenpair derivatives for asymmetric damped systems with distinct and repeated eigenvalues is presented in [25,26]. A very simple and efficient method is also presented by Lee and Jung [27,28] for the computation of eigenpair derivatives for symmetric eigenvalue problems. The method finds the eigenvector derivative by solving an algebraic equation with a symmetric coefficient matrix.

The major contributions of this paper are as following: Firstly, it is revised that the eigenpair derivatives can be used in the perturbation analysis for approximating the eigenpairs of a perturbed system. Secondly, based on the direct methods for the calculation of the eigenpair derivatives, an iterative algorithm is developed to efficiently calculate the eigenpairs of a perturbed structure. This is the most important part of the paper. Finally, the equivalence of the first-order numerical approximation of the eigenfrequency with the Slater's perturbation theorem, as an analytical counterpart, is shown.

This paper is organized as follows: Section 2 describes the transfer of the Helmholtz equation to the generalized eigenvalue problem by means of the finite element method (FEM). Section 3 explains the approximation of perturbed eigenpairs by eigenpair derivatives. This section is composed of two parts: In Subsection 3.1 the modal superposition method for the calculation of the eigenpair derivative is presented and the drawbacks are briefly discussed. In Subsection 3.2 an iterative algebraic algorithm for the calculation of perturbed eigenpairs is developed. The modal superposition method and proposed iterative algorithm are tested on a circular waveguide in Subsection 4.1 and 4.2.1 respectively. The iterative algebraic method is also tested on an accelerating cavity model as an application example in Subsection 4.2.2 to illustrate the accuracy and efficiency of the proposed method. In the appendix of this paper, the analytical and numerical perturbation methods are briefly compared.