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An asymptotic preserving multidimensional ALE method for a system of two compressible flows coupled with friction

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ABSTRACT

We present a multidimensional asymptotic preserving scheme for the approximation of a mixture of compressible flows. Fluids are modelled by two Euler systems of equations coupled with a friction term.
The asymptotic preserving property is mandatory for this kind of model, to derive a scheme that behaves well in all regimes (*i.e.* whatever the friction parameter value is). The method we propose is defined in ALE coordinates, using a Lagrange plus remap approach. This imposes a multidimensional definition and analysis of the scheme.

1. Introduction

A multifluid model is a model for a fluid mixture for which each fluid is described by its own full set of variables (for instance density, velocity and energy). The model is generally closed in a way that defines interactions between the constituents, depending on the involved physics. These models are widely used in different communities. One very popular model of this kind is the Baer–Nunziato model [3] for deflagration-to-detonation transition of reactive flows. Many numerical methods to approximate this model have been designed, we refer to a few of them [37,14,15,1,2]. Scannapieco and Cheng [38] also derive similar kind of model for turbulent flows and apply it to describe a mixing zone driven by Rayleigh–Taylor or Richtmyer–Meshkov instabilities [12]. Such kind of model is also used in plasma physics to account for plasmas collision or Non-Local-Thermodynamic-Equilibrium (NLTE) Ion–Electron interactions [17,39]. Although all the analysis done in this paper can be applied to any of the former models, we are in particular interested in the latter application. In this context, multifluid models are a good approximation, in particular to account for the collision of two ion populations, each of them being at LTE. However, to our knowledge, these models are never used for plasma collisions. The reason for this is stated by R. Sentis in [39]: "*The [...] system may be quite difficult to solve in two- or three-dimensional geometry, especially in the case when the friction coefficient [...] is large [...].*" Consequently, a simplified model is in general preferred, in which the velocity gap between the two fluids is modelled by a diffusion process on the concentrations. Unfortunately, it implies empirical closures and exhibits bad behaviour at high temperatures (when the coupling between the ion populations is weak).

In the following, we explain why the classical schemes for the multifluid system fail to capture the strong coupling limit. It is in fact inherent to this kind of model and relies to the asymptotic preserving (AP) property [28–31] in the high friction

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regime or infinite friction regime. In the former regime, the fluids interpenetration follows a diffusion law. In the latter one, the mixture evolves as a single fluid, see (4)–(5). If no attention is paid to these regimes, the scheme will fail to capture it at a reasonable calculation cost. Some authors [13,16,23] propose an asymptotic discretization for the system (1) in 1D in the Eulerian frame — multidimensional calculations being achieved by means of directional splitting —, but no asymptotic preserving scheme has been yet published for 2D unstructured meshes for this model. A similar ALE formalism is used to treat multifluid interaction in [11]. Authors use the Compatible Hydro scheme [8] and do not analyze the asymptotic preserving property since they mainly focus on the physics of the coupling.

In this paper, we propose a multidimensional scheme to approximate solutions of this kind of model, written in (1), which captures accurately the asymptotic regime. We want our scheme to be able to deal with Arbitrary-Lagrange–Euler (ALE) frame and unstructured meshes in order to properly handle highly deformed calculation domains. Even for simpler models, only few unstructured asymptotic preserving schemes have been developed (refer for instance to Berthon and Turpault [5] and Franck *et al.* [7,24]). The scheme we propose in Section 4 has connections with [25,26], where an Euler with friction system is studied in the limit of high friction for long time, providing a different kind of scaling. So, the proposed scheme is not a direct extension of [25] to the bi-fluid case. The scheme presented in this work is split into two steps. In the first step we solve two Euler systems of equations coupled by friction. Since each fluid has its own velocity, the Lagrangian mesh of each fluid will evolve separately during this step. Then, in the second step, the conservative variables vector of each of the fluids will be projected onto a common mesh (not necessarily identical to the initial mesh).

In the Section 2 of this paper, we recall the properties of the model we consider, that are conservation, hyperbolicity, and asymptotic limit model. In Section 3, we recall the basis of the solver (Glace [10] or Eucclhyd [35]) used to compute the Lagrangian step. The Section 4 describes the Lagrangian step of the proposed scheme. It is demonstrated that the scheme preserves the properties of conservation, stability and consistency with respect to the continuous model for all regimes (independently of the value of the friction parameter). Then in Section 5, our ALE strategy is described. Finally, Section 6 is devoted to numerical experiments on several problems (Sod shock tube, triple point and Rayleigh–Taylor). Some comparisons with a non-AP scheme are provided.

2. A two fluids model with friction

Let us consider a mixture of two fluids f_1 and f_2. In the following, we will denote by "multi-fluid model", a model for which each fluid $\alpha \in \{f_1, f_2\}$ is represented by its own set of variables: $(\rho^\alpha, \mathbf{u}^\alpha, E^\alpha)$. Conversely, we will refer as "mono-fluid model", a model describing a mixture where mean quantities are considered $(\rho, \mathbf{u}, E)$, each fluid position being precised by an additional equation on the concentration (e.g. $\chi := \frac{\rho^\alpha}{\rho^\alpha+\rho^\beta}$).

In this part, we present a simplified version of Scannapieco–Cheng's model where the interaction between the two constituents reduces to a friction term. In semi-Lagrangian coordinates, for each fluid $\alpha \in \{f_1, f_2\}$ (β denoting the other fluid), the model reads

$$\begin{aligned}
&\rho^\alpha D_t^\alpha \tau^\alpha = \nabla \cdot \mathbf{u}^\alpha,\\
&\rho^\alpha D_t^\alpha \mathbf{u}^\alpha = -\nabla p^\alpha - \nu\rho\delta\mathbf{u}^\alpha,\\
&\rho^\alpha D_t^\alpha E^\alpha = -\nabla\cdot(p^\alpha \mathbf{u}^\alpha) - \nu\rho\delta\mathbf{u}^\alpha\cdot\overline{\mathbf{u}},
\end{aligned} \tag{1}$$

where $\rho^\alpha, \mathbf{u}^\alpha$ and E^α respectively denote the mass density, the velocity and the total energy density of fluid α. Also, $\tau^\alpha = \frac{1}{\rho^\alpha}$ denotes the specific volume. The pressure p^α satisfies the equation of state $p^\alpha := p^\alpha(\rho^\alpha, e^\alpha)$, where e^α, the internal energy density, is defined by $e^\alpha := E^\alpha - \frac{1}{2}\|\mathbf{u}^\alpha\|^2$. The total density ρ and the mean velocity $\overline{\mathbf{u}}$ are defined as $\rho := \rho^\alpha + \rho^\beta$ and $\rho\overline{\mathbf{u}} := \rho^\alpha\mathbf{u}^\alpha + \rho^\beta\mathbf{u}^\beta$. The term $\delta\mathbf{u}^\alpha$ is the velocity difference, the $\delta(\cdot)^\alpha$ operator being defined by $\delta\phi^\alpha = -\delta\phi^\beta = \phi^\alpha - \phi^\beta$. Finally, ν is the friction parameter. Also, remark that the Lagrangian derivative $D_t^\alpha := \partial_t + \mathbf{u}^\alpha\cdot\nabla$, is obviously not the same for each fluid.

The entropy η^α defined by Gibbs formula $T^\alpha d\eta^\alpha = de^\alpha + p^\alpha d\tau^\alpha$ satisfies the following entropy inequality

$$T^\alpha D_t^\alpha \eta^\alpha \geq \nu\frac{\tau^\alpha}{\tau^\beta}\delta\mathbf{u}^\alpha\cdot\delta\mathbf{u}^\alpha \geq 0. \tag{2}$$

Prior to establishing a numerical scheme that discretizes this set of six equations, we recall some properties of the model itself.

Property 1 *(Conservation). The model (1) is conservative in volume and mass for each fluid. Also, it is conservative in the sum of momenta and in the sum of the total energies of the two fluids.*

Proof. Conservation of mass and volume is obvious since the first equation of (1) is the continuity equation written for each fluid.

Conservation of momenta sum and total energies sum require more cautiousness, since Lagrangian derivative are not the same for each fluid. To establish them one rewrites (1) in an Eulerian framework.