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A simple, robust and efficient high-order accurate shock-capturing scheme for compressible flows: Towards minimalism

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ABSTRACT

Developed is a high-order accurate shock-capturing scheme for the compressible Euler/Navier–Stokes equations; the formal accuracy is 5th order in space and 4th order in time. The performance and efficiency of the scheme are validated in various numerical tests. The main ingredients of the scheme are nothing special; they are variants of the standard numerical flux, MUSCL, the usual Lagrange's polynomial and the conventional Runge–Kutta method. The scheme can compute a boundary layer accurately with a rational resolution and capture a stationary contact discontinuity sharply without inner points. And yet it is endowed with high resistance against shock anomalies (carbuncle phenomenon, post-shock oscillations, etc.). A good balance between high robustness and low dissipation is achieved by blending three types of numerical fluxes according to physical situation in an intuitively easy-to-understand way. The performance of the scheme is largely comparable to that of WENO5-Rusanov, while its computational cost is 30–40% less than of that of the advanced scheme.

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1. Introduction

The history of shock capturing dates back as far as 1944, when John von Neumann wrote a proposal of a numerical method for flows with shock waves [1]. His proposal was immediately adopted in Manhattan project but it was in 1948, three years after the atomic bombings of Hiroshima and Nagasaki, when the goal was achieved by Robert Richtmyer, though not very satisfactorily in the light of today's standard (see his Los Alamos technical reports [2,3]). Richtmyer's open and honest words "mock dissipative terms" [3] seem to hit something like the essence of shock capturing. Incidentally, these milestone reports were classified without any rational reason until 1993. The interested reader is referred to e.g. [4] and references therein for the history of shock-capturing schemes in the early days.

A gas is highly nonequilibrium inside a nonweak shock wave. Its thickness is of the order of the mean free path of gas molecules (about 10^{-8} m under standard conditions for temperature and pressure), which is, generally speaking, much smaller than the Kolmogorov length, i.e. the smallest scale in turbulent flows. The orthodox description of its internal structure requires kinetic theory (the Boltzmann equation). It is, however, too ambitious to try to perform a numerical simulation of a high-speed flow past a body, such as a 10 meter-long hypersonic technology vehicle, which employs a kinetic equation in the vicinities of shock waves formed around the airframe with a resolution sufficient for accurate approximation

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of their internal structures. The situation does not change at all even if the laborious kinetic equation is renounced and is replaced by the inappropriate but effortless compressible Navier–Stokes (NS) equations. Shock capturing is far from the pursuit of physical reality, which requires both correct physical description and a sufficiently high resolution. Rather, it is a compromising or conciliatory technique to automatically capture a shock wave within the range of a few mesh intervals, which is many orders of magnitude larger than its real thickness, without the occurrence of spurious oscillations and the violation of the second law of thermodynamics. The rationality of this inexpensive and defensive computation is not lost at all even nowadays, when computer technology has significantly developed and the necessity of multi-scale analyses is loudly claimed in many fields of science and technology.

Theoretical studies on shock capturing have brought about several bases of numerical methods for hyperbolic PDEs. Most of secure theories for shock capturing, however, are limited to the case of scalar conservation laws even now. That is, it follows that empiricism is still indispensable for shock capturing in engineering problems. One of the typical examples is the mitigation of shock anomalies such as carbuncle phenomenon and post-shock oscillations, which arise in hypersonic flow regime and considerably deteriorate the quality and reliability of outcomes (see e.g. [5,6] and references therein), and there is still no theoretically justified prescription for the suppression of these unfavorable pathologies. Under such situations, it is not necessarily meaningless to go back to the spirit of shock capturing in the early days and try to make the modern shock capturing as easy as possible. Such attempts are expected to yield at least simple schemes for confirmation of the authenticity of existing or future advanced shock-capturing technologies. Even in case where such stalking horses are beyond the expectation, our understanding of shock capturing ought to be deepened. Indeed, such simple but capable schemes are desired in various fields of engineering and they can also be considered as springboards of future advanced shock-capturing schemes.

In the present paper, we try to build a high-end shock-capturing scheme for the compressible Euler/NS equations out of elementary or generic materials and gadgets, following the above-mentioned spirit. The specifications of the scheme are as follows.

- **i)** It is categorized into the finite volume method.
- **ii)** It is not only shock capturing but also boundary-layer resolving; a boundary layer is accurately computed with a rational resolution such as five mesh points in the layer.
- **iii)** It is sharp contact-discontinuity capturing; a stationary contact discontinuity is captured without inner points.
- **iv)** Its formal accuracy is 5th order in space and 4th order in time.
- **v)** It is computationally efficient.
- **vi)** It is endowed with high resistance against shock anomalies.

The challenge in the present study is to balance the low dissipation and the high robustness.

The rest of the paper is organized as follows. In Sec. 2, two formulas of the numerical flux are presented. One is dissipative and the other is less dissipative. The former takes charge of shock capturing and the latter does of regions away from shock waves. These formulas are simplified and one of them is further modified for sharp contact-discontinuity capturing. In Sec. 3, two reconstructions are reviewed. One is MUSCL with van Leer slope limiter and the other is the conventional 5th order accurate approximation using quartic functions generated from the primitive functions of the conservative variables. Three numerical fluxes are computed by means of these generic methods and are blended according to physical situation in an intuitively easy-to-understand way (Sec. 4). Before the validation of the scheme built in Sec. 4, an example which shows the rationality of shock capturing in an extreme situation is presented (Sec. 5); the problem of the interaction between a very small vortex and a normal shock wave is numerically analyzed on the basis of the BGK equation and the numerical solution is compared with that of a shock-capturing scheme. The performance and efficiency of the present scheme are tested in Sec. 6. Concluding remarks are made in Sec. 7. Some technical details in the 2D computation are described in Appendix.

2. Numerical flux

The kinetic shock-capturing scheme of [7] is employed as the springboard of the scheme that we are going to build here. This scheme is robust and boundary-layer resolving. Although the numerical fluxes are derived via kinetic theory, however, kinetic theory itself is not essential for the above-mentioned characteristics. In order to elucidate it, they are explained as variants of the standard numerical flux.

2.1. Notation and basic equations

The gas is assumed to be ideal for simplicity. Time and Cartesian space coordinates are denoted by t and (x, y, z), respectively. Suppose that we are going to numerically solve an equation system in the form

$$\frac{\partial \mathcal{K}}{\partial t} + \frac{\partial F(\mathcal{K})}{\partial x} + \frac{\partial G(\mathcal{K})}{\partial y} + \frac{\partial H(\mathcal{K})}{\partial z} = 0,$$

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