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An isogeometric boundary element method for electromagnetic scattering with compatible B-spline discretizations

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ABSTRACT

We outline the construction of compatible B-splines on 3D surfaces that satisfy the continuity requirements for electromagnetic scattering analysis with the boundary element method (method of moments). Our approach makes use of Non-Uniform Rational B-splines to represent model geometry and compatible B-splines to approximate the surface current, and adopts the isogeometric concept in which the basis for analysis is taken directly from CAD (geometry) data. The approach allows for high-order approximations and crucially provides a direct link with CAD data structures that allows for efficient design workflows. After outlining the construction of div- and curl-conforming B-splines defined over 3D surfaces we describe their use with the electric and magnetic field integral equations using a Galerkin formulation. We use Bézier extraction to accelerate the computation of NURBS and B-spline terms and employ $\mathscr{H}$-matrices to provide accelerated computations and memory reduction for the dense matrices that result from the boundary integral discretization. The method is verified using the well known Mie scattering problem posed over a perfectly electrically conducting sphere and the classic NASA almond problem. Finally, we demonstrate the ability of the approach to handle models with complex geometry directly from CAD without mesh generation.

1. Introduction

Research into unifying geometry and analysis for efficient design workflows has progressed rapidly in recent years driven by the isogeometric analysis and computational geometry research communities. Analysis based on geometry discretizations now covers a wide range of technologies including NURBS [1], T-splines [2], LR B-splines [3], PHT-splines [4] and subdivision surfaces [5]. A major research challenge at present is the automatic generation of volumetric discretizations from given geometric surface data and promising research includes the work of [6,7] based on T-splines. In contrast, analysis methods based on shell formulations or boundary integral methods are known to require only a surface discretization exhibiting key benefits for a common geometry and analysis model since no additional volumetric processing is required. There has been

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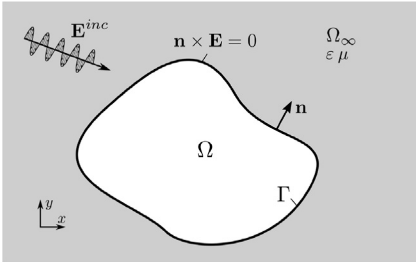

Fig. 1. A PEC domain residing within an infinite domain impinged by an electromagnetic plane wave.

much research into isogeometric shell formulations including [5,8,9] and developments into isogeometric boundary element methods based on NURBS [10,11], T-splines [12,13] and subdivision surfaces [14].

A key application of the boundary element method is the analysis of electromagnetic scattering over complex geometries in which a perfectly electrically conducting (PEC) assumption can be made. The method is often termed the method of moments within the electromagnetic research community but is synonymous with the Galerkin boundary element method. It is well known that a straightforward application of nodal basis functions to the electric and magnetic field integral equations (EFIE, MFIE) prevents numerical convergence and instead, discrete spaces that satisfy the relevant continuity requirements must be used. The most commonly used discretization that satisfies the relevant continuity requirements are Raviart–Thomas [15] or RWG [16] basis functions that are mainly based on low order polynomials.

In the context of isogeometric analysis progress has been made on the development of spline-based compatible discretizations [17–20] in which a discrete de Rham sequence can be constructed providing a crucial step towards application of isogeometric analysis for fluid flow and electromagnetics applications. This fundamental work opens up the opportunity for the development of an isogeometric boundary element method (isogeometric method of moments) for electromagnetic scattering which is the focus of the present study. We note similar work in which subdivision surfaces are employed [21], but we believe that use of B-spline based algorithms provides greater refinement flexibility, provide a natural link with NURBS based systems that are ubiquitous in modern engineering design software, and offer higher convergence rates over equivalent subdivision schemes with extraordinary points.

We organize the paper as follows: first, we prescribe the Galerkin formulation of the relevant integral equations that govern electromagnetic scattering; we give an overview of NURBS surfaces and detail the construction of compatible B-splines; we then specify the fully discretized form of the integral equations for electromagnetic scattering with compatible B-splines; we cover implementation details of the method including fast evaluation of basis functions through Bézier extraction and the use of $\mathscr{H}$-matrices to approximate dense matrices; we verify the present method by performing electromagnetic scattering over a sphere in which a closed-form solution is provided by Mie scattering theory and finally, we demonstrate the ability of the present approach to perform electromagnetic scattering of PEC bodies with complex geometries taken directly from CAD software. It is assumed that time-harmonic fields are prescribed and, unless stated otherwise, it can be assumed that $\mathbf{x}\in\mathbb{R}^3$.

2. Electric field integral equation: Galerkin formulation

We first assume a PEC domain Ω with connected boundary $\Gamma := \partial\Omega$ residing within an unbounded domain Ω_∞ with isotropic t permittivity and permeability given by the scalar quantities ε and μ respectively. We further assume a polarized time-harmonic electromagnetic plane wave of angular frequency ω is imposed on the PEC body with a wavenumber $k = \omega\sqrt{\varepsilon\mu}$. Denoting $\mathbf{E}$ as the total electric field, in the presence of an electromagnetic wave a surface current $\mathbf{J}$ is induced and the following PEC condition holds on the surface of the scattered object

$$\mathbf{n}\times\mathbf{E}=0 \tag{1}$$

where $\mathbf{n}$ represents the outward pointing normal vector. We specify the incident wave as $\mathbf{E}^i(\mathbf{x}) = \mathbf{p}e^{-jk\mathbf{d}\cdot\mathbf{x}}$ where j is the unit imaginary number, $\mathbf{p} = (p_x, p_y, p_z)$ is a polarization vector and $\mathbf{d} = (d_x, d_y, d_z)$, $|\mathbf{d}| = 1$ is a propagation vector. The relationship between the total, incident and scattered electric fields is written as

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s \tag{2}$$

where $\mathbf{E}^s$ represents the scattered electric field. The entire set-up is depicted in Fig. 1.

Following the potential formulation of Maxwell's equations (see e.g. [22]), the scattered electric field can be expressed in terms of an electric potential φ and magnetic vector potential $\mathbf{A}$ (assuming time-harmonic fields) as

$$\mathbf{E}^s = -j\omega\mathbf{A} - \nabla\varphi \tag{3}$$

where the electric potential is given by